MMDS
Center for Mathematical Modeling and Data Science
Osaka University

Workshop on "Mathematical finance and related issues"

March 13–16, 2018
Osaka University Nakanoshima Center

Program and Abstracts

Organized through a joint collaboration of
Center for Mathematical Modeling and Data Science, Osaka University
Research Center for Quantitative Finance, Tokyo Metropolitan University
Grants-in-Aid for Scientific Research (KAKENHI) of Japan Society for the Promotion of Science (JSPS)

大阪大学
OSAKA UNIVERSITY

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Peter Friz (Technische Universität Berlin)
Masaaki Fukasawa (Osaka University / Tokyo Metropolitan University)
Yuzuru Inahama (Kyushu University)
Hideo Nagai (Kansai University)
Jun Sekine (Osaka University)

Local Organizers

Masaaki Fukasawa (Osaka University / Tokyo Metropolitan University)
Nobuaki Naganuma (Osaka University)
Jun Sekine (Osaka University)
Dai Taguchi (Osaka University)

Practical Information

- Venue: Osaka University Nakanoshima Center

  4-5-53 Nakanoshima, Kita-ku, Osaka 530-0005 Japan.
  Tel: +81 (0)6 6444 2100 Fax: +81 (0)6 6444 2338

- Welcome Party: March 13 (Tuesday), 18:00 – 20:00

  Salon de L'Amical, 9th floor of the Nakanoshima Center.

- Conference Dinner: March 15 (Thursday), 18:00 – 20:00

  MINAMO, 12th floor of Nakanoshima Plaza, Tel: 06-6449-6901.

Workshop on "Mathematical finance and related issues"

March 13–16, 2018, Osaka University Nakanoshima Center, Osaka, Japan

Program

Tuesday, March 13

09:30 – 09:40 Opening

March 13, 09:40 – 12:00

09:40 – 10:20 Terry Lyons (University of Oxford)
TBA

10:20 – 11:00 Yu Ito (Kyoto Sangyo University)
A fractional calculus approach to rough path integration

11:00 – 11:20 Coffee Break

11:20 – 12:00 Yuzuru Inahama (Kyushu University)
Short time full asymptotic expansion of hypoelliptic heat kernel at the cut locus

12:00 – 14:00 Lunch Break

March 13, 14:00 – 17:20

14:00 – 14:40 Shigeki Aida (The University of Tokyo)
Rough differential equations containing path-dependent bounded variation terms

14:40 – 15:20 Ismael Bailleul (Institut de recherche mathematiques de Rennes)
Mean field rough differential equations

15:20 – 16:00 Coffee Break

16:00 – 16:40 David Prömel (University of Oxford)
Optimal extension to Sobolev rough paths

16:40 – 17:20 Thomas Cass (Imperial College London)
Tree algebras over topological vector spaces in rough path theory

18:00 – 20:00 Welcome Party

Wednesday, March 14

March 14, 09:40 – 12:00

09:40 – 10:20 Shigeo Kusuoka (The University of Tokyo)
Euler-Maruyama Approximation and Greeks

10:20 – 11:00 Yuji Shinozaki (Tokyo Institute of Technology / The bank of Tokyo-Mitsubishi UFJ)
Higher order discretization methods of forward-backward SDEs using KLNV-scheme : Application to XVA pricing

11:00 – 11:20 Coffee Break

11:20 – 12:00 Toshihiro Yamada (Hitotsubashi University)
Higher order discretization methods using Malliavin Monte Carlo and Brownian Markov chain without Lévy area simulation

12:00 – 14:00 Lunch Break

14:00 – 17:20 Discussions

Thursday, March 15

March 15, 09:40 – 12:00

09:40 – 10:20 Jim Gatheral (Baruch College of The City University of New York)
Diamonds: A quant's best friend

10:20 – 11:00 Paul Gassiat (Université Paris Dauphine)
A regularity structure for rough volatility

11:00 – 11:20 Coffee Break

11:20 – 12:00 Elisa Alòs (Universitat Pompeu Fabra)
Estimating the Hurst parameter from short term volatility swaps

12:00 – 14:00 Lunch Break

March 15, 14:00 – 17:20

14:00 – 14:40 Peter Friz (Technische Universität Berlin / Weierstrass Institute for Applied Analysis and Stochastics)
From rough paths and regularity structures to short dated option pricing under rough volatility

14:40 – 15:20 Stefan Gerhold (Vienna University of Technology)
Moment Explosions in the Rough Heston Model

15:20 – 16:00 Coffee Break

16:00 – 16:40 Antoine Jacquier (Imperial College London)
TBA

16:40 – 17:20 Tai-ho Wang (Baruch College of The City University of New York)
Target volatility option pricing in lognormal fractional SABR model

18:00 – 20:00 Conference Dinner

Friday, March 16

March 16, 09:40 – 12:00

09:40 – 10:20 Shige Peng (Shandong University)
BSDE and Reflected BSDE driven by G-Brownian Motion

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Malliavin calculus for Dyson Brownian motions

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Rough Volatility and No Arbitrage

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Functional central limit theorems for non-symmetric random walks on nilpotent covering graphs

14:40 – 15:20 Coffee Break

16:00 – 16:40 Arturo Kohatsu-Higa (Ritsumeikan University)
Unbiased simulation methods of order two

16:40 – 16:50 Closing

TBA

Terry Lyons (University of Oxford)

March 13, 09:40 – 10:20

A fractional calculus approach to rough path integration

Yu Ito (Kyoto Sangyo University)

March 13, 10:20 – 11:00

This study is an alternative approach to the fundamental theory of rough paths on the basis of fractional calculus. In this talk, using fractional calculus, we will provide an expression of the rough path integral by M. Gubinelli (2004). The expression is given explicitly by the Lebesgue integrals for fractional derivatives. This also provides such an expression of the rough path integral by T. J. Lyons (1998) for β-Hölder rough paths with $\beta \in (1/3, 1/2]$. Our result is a generalization of that of Y. Hu and D. Nualart (2009), and our proof is based on a method by M. Zähle (1998).

Short time full asymptotic expansion of hypoelliptic heat kernel at the cut locus

Yuzuru Inahama (Kyushu University)

March 13, 11:20 – 12:00

In this talk we prove a short time asymptotic expansion of a hypoelliptic heat kernel on an Euclidean space and a compact manifold. This kind of problem is traditional and very important in analysis, probability and geometry, but it also exists in math finance, too. We study the "cut locus" case, namely, the case where energy-minimizing paths which join the two points under consideration form not a finite set, but a compact manifold. Under mild assumptions we obtain an asymptotic expansion of the heat kernel up to any order. Our approach is probabilistic and the heat kernel is regarded as the density of the law of a hypoelliptic diffusion process, which is realized as a unique solution of the corresponding stochastic differential equation. Our main tools are S. Watanabe's distributional Malliavin calculus and T. Lyons' rough path theory.

This is a joint work with Setsuo Taniguchi (Kyushu University).

Rough differential equations containing path-dependent bounded variation terms

Shigeki Aida (The University of Tokyo)

March 13, 14:00 – 14:40

In the framework of Itô's stochastic calculus, path-dependent stochastic differential equations (=SDEs) driven by Brownian motions can be defined and solved uniquely under the bounded and Lipschitz condition of the coefficient with respect to the uniform convergence topology. Stochastic differential delay equations are typical examples of such equations. Reflected SDEs on domains also can be viewed as path-dependent SDEs by using the Skorohod map. If the domain is a half space of an Euclidean space, then the Skorohod map is a Lipschitz map. Accordingly, the coefficient of the corresponding path-dependent SDE satisfies the Lipschitz condition and the existence and the uniqueness of the solution holds true. Since the smooth domain is locally diffeomorphic to the half space, we can prove the existence and the uniqueness of the solution to the reflected SDEs. However, the Skorohod map is not Lipschitz for general domain which are studied by Tanaka, Lions and Sznitman, Saisho. In that case, the unique existence of the strong solution to the reflected SDE is proved by proving the pathwise uniqueness of the SDE and the path-dependent SDE is not used for such problems. In this talk, we discuss the existence of solutions to a class of path-dependent rough differential equations which include reflected SDEs.

Mean field rough differential equations

Ismael Bailleul (Institut de recherche mathematiques de Rennes)

March 13, 14:40 – 15:20

Mean field type stochastic equations provide a universal description of a whole class of very large systems of interacting dynamics, by describing the behaviour of a 'typical particle' in the system as driven by its own law, rather than by the environment provided by the other particles of the system. Motivated by questions on Boltzmann equation, the study of mean field type stochastic differential equations was the subject of intense activities in the eighties. The realm of It calculus is not sufficient to cover all the natural phenomena of interest, though, and one had to wait for a seminal paper by Cass and Lyons on mean field type rough differential equations (2010) to get a first work that goes beyond the It setting. They proved well-posedness for an equation with linear mean field interaction in the drift only and proved propagation of chaos for the associated particle system. This result was extended shortly afterward (2012) with a well-posedness result for an equation with nonlinear mean field interaction in the drift only and by proving an existence result for equation with mean field interaction both in the drift and diffusivity. The problem of proving well-posedness of the equation in the latter case remained opened so far. I will explain how this can be done via the introduction of a new rough path-like setting and an associated notion of controlled path, with the crucial input of Lions' differential calculus on Wasserstein space.

Optimal extension to Sobolev rough paths

David Prömel (University of Oxford)

March 13, 16:00 – 16:40

We show that every n-dimensional path with sufficient Sobolev regularity can be lifted in a unique, optimal and deterministic way to a Sobolev rough path in the sense of T. Lyons. In addition, we prove that the solution map associated to differential equations driven by rough paths is locally Lipschitz continuous w.r.t. the Sobolev topology on the rough path space for any arbitrary low regularity. The talk is based on a joint work (in progress) with Chong Liu (ETH Zurich) and Josef Teichmann (ETH Zurich).

Tree algebras over topological vector spaces in rough path theory

Thomas Cass (Imperial College London)

March 13, 16:40 – 17:20

Euler-Maruyama Approximation and Greeks

Shigeo Kusuoka (The University of Tokyo)

March 14, 09:40 – 10:20

Euler-Maruyama Approximation is widely used in financial firms to compute prices of derivatives, and its convergence rate has been studied by many authors. But people in financial firms also use Euler-Maruyama approximation and its difference to compute Greeks on price of derivatives. In this paper, we discuss how accurate this method to compute Greeks is.

Higher order discretization methods of forward-backward SDEs using KLNV-scheme : Application to XVA pricing

Yuji Shinozaki[1] (Tokyo Institute of Technology/The bank of Tokyo-Mitsubishi UFJ)

March 14, 10:20 – 11:00

New higher order discretization methods of forward-backward stochastic differential equations (FBSDEs) will be presented. In particular, the solution (X_s, Y_s) with the natural filtration $\mathcal{F}_s$ of the following equation:

$$X_s = x + \sum_{i=0}^{d} \int_t^s V_i(X_u) \circ dB^i(u),$$

$$Y_s = \Phi(X_T) + \int_s^T f(X_u, Y_u)du - \sum_{i=0}^{d} \int_s^T Z_u^{(i)} dB^i(u),$$

where $x \in \mathbb{R}^N$, $V_0, V_1, \ldots, V_d \in C_b^\infty(\mathbb{R}^N; \mathbb{R}^N)$ and $\Phi : \mathbb{R}^N \to \mathbb{R}$, $f : \mathbb{R}^{N+1} \to \mathbb{R}$ are appropriately smooth functions is considered. The aim of this talk is to propose a method for numerical calculation of $E[Y_t \mid \mathcal{F}_t]$ by using the simulation. Our approach is based on higher order discretization methods of FBSDEs; namely, sets of random variables $\{\hat{X}_{s_i}, \hat{Y}_{s_i}\}_{i=0,1,\ldots n}$ $(t = s_0 < s_1 \cdots < s_n = T)$ which satisfy $\left|E[Y_t \mid \mathcal{F}_t] - E[\hat{Y}_t \mid \hat{\mathcal{F}}_t]\right| < C_p/n^p,\ p \leq 2$ for any n (order p method), will be constructed. In these proposed methods, the forward component is discretized using the KLNV-scheme [4] with discrete random variables, and the backward component using a higher order numerical integration method suitable for the discretization method of the forward component; specifically, the Ninomiya–Victoir [7] and Ninomiya–Ninomiya [5] methods combined with the trapezoidal rule attain the second order and the $Q_{(s)}^{(7,2)}$ method [8] with Simpson's the third order. To conquer the nested simulation or multiple Monte Carlo problems, the tree based branching algorithm technique [1, 6] is used in the same way as [2, 3]. The authors also apply the proposed methods to the XVA pricing, particularly to the credit valuation adjustment (CVA). The proposed methods calculate CVA prices formulated by FBSDEs which are usually considered to be hard to solve numerically. The numerical results show that the expected theoretical order and computational efficiency are achieved.

Joint work with Syoiti Ninomiya, Tokyo Institute of Technology.

References

[1] Dan Crisan and Terry Lyons. Minimal entropy approximations and optimal algorithms. *Monte Carlo Methods and Applications*, 8(4):343356, 2002.

[2] Dan Crisan and Konstantinos Manolarakis. Solving backward stochastic differential equations using the cubature method: Application to nonlinear pricing. *SIAM Journal on Financial Mathematics*, 3(1):534571, 2012.

[1] The views expressed in this talk are those of the author and do not necessarily reflect the official views of the affiliated institutions of the author.

[3] Dan Crisan and Konstantinos Manolarakis. Second order discretization of backward SDEs and simulation with the cubature method. *Annals of Applied Probability*, 24(2):652678, 2014.

[4] Shigeo Kusuoka. Approximation of expectation of diffusion process and mathematical finance. *Proceedings of Final Taniguchi Symposium, Nara, volume 31 of Advanced Studies in Pure Mathematics*, pages 147165, 2001.

[5] Mariko Ninomiya and Syoiti Ninomiya. A new higher-order weak approximation scheme for stochastic differential equations and the RungeKutta method. *Finance and Stochastics*, 13(3):415443, 2009.

[6] Syoiti Ninomiya. A partial sampling method applied to the Kusuoka approximation. *Monte Carlo methods and Applications*, 9(1):2738, 2003.

[7] Syoiti Ninomiya and Nicolas Victoir. Weak approximation of stochastic differential equations and application to derivative pricing. *Applied Mathematical Finance*, 15(2):107121, 2008.

[8] Yuji Shinozaki. Construction of a third-order K-scheme and its application to financial models. *SIAM Journal on Financial Mathematics*, 8(1):901932, 2017.

Higher order discretization methods using Malliavin Monte Carlo and Brownian Markov chain without Lévy area simulation

Toshihiro Yamada (Hitotsubashi University)

March 14, 11:20 – 12:00

The aim of this talk is to show higher order weak approximations of SDEs and their applications. We introduce a general discretization method using Malliavin Monte Carlo simulation. Second and third order schemes are explicitly obtained and a hypoelliptic extension is shown with numerical examples. Further, we give a high order discretization method for the Bismut-Elworthy-Li formula and Greeks as an application. We also show a second order scheme for SDEs using a Markov chain constructed by second order polynomials of Brownian motions. Any discrete moment matched random variables or Lévy area simulation method are not used but a correction term with Lie bracket of vector fields associated with SDE appears in the Markov chain. The effectiveness is checked through numerical experiments.

Diamonds: A quant's best friend

Jim Gatheral (Baruch College of The City University of New York)

March 15, 09:40 – 10:20

We use the Alòs Itô Decomposition Formula to express certain conditional expectations as exponentials of forests of trees. Each tree represents iterated applications of a new diamond operator. As applications, we compute exact formal expression for stochasticity, the leverage swap, and skewness for any stochastic volatility model expressed in forward variance form. As yet another application, we show how to extend the Bergomi-Guyon expansion to all orders in volatility of volatility. Finally, we use the forest expansion to compute a closed-form expression for the leverage swap in the rough Heston model and give an example of its use for fast calibration.

A regularity structure for rough volatility

Paul Gassiat (Université Paris Dauphine)

March 15, 10:20 – 11:00

Estimating the Hurst parameter from short term volatility swaps

Elisa Alòs (Universitat Pompeu Fabra)

March 15, 11:20 – 12:00

This work is devoted to studying the difference between the fair strike of a volatility swap and the at-the-money implied volatility (ATMI) of a European call option. It is well-known that the difference between these two quantities converges to zero as the time to maturity decreases. In this work, we make use of a Malliavin calculus approach to derive an exact expression for this difference. This representation allows us to establish that the order of the convergence is different in the correlated and in the uncorrelated case, and that it depends on the behavior of the Malliavin derivative of the volatility process. In particular, we will see that for volatilities driven by a fractional Brownian motion, this order depends on the corresponding Hurst parameter H. Moreover, in the case $H \geq 1/2$, we develop a model-free approximation formula for the volatility swap, in terms of the ATMI and its skew.

Joint work with Kenichiro Shiraya, The University of Tokyo.

From rough paths and regularity structures to short dated option pricing under rough volatility

Peter Friz (Technische Universität Berlin / Weierstrass Institute for Applied Analysis and Stochastics)

March 15, 14:00 – 14:40

We consider rough stochastic volatility models. Specifically, volatility has fractional - worse than diffusion - scaling, a regime which recently attracted considerable attention both from the statistical and option pricing point of view. With focus on the latter, we sharpen the large deviation result of Forde-Zhang in a way that allows to zoom-in around the money while maintaining full analytical tractability. Mathematically speaking, this amounts to prove higher order moderate deviations estimates, recently introduced in the option pricing context by Friz, Gerhold and Pinter. We also point out some uses of rough path ideas. (Joint work with Christian Bayer, Paul Gassiat, Archil Gulisashvili, Blanka Horvath, Jrg Martin, Benjamin Stemper.)

Moment Explosions in the Rough Heston Model

Stefan Gerhold (Vienna University of Technology)

March 15, 14:40 – 15:20

We show that the moment explosion time in the rough Heston model [El Euch, Rosenbaum 2016, arxiv:1609.02108] is finite if and only if it is finite for the classical Heston model. Upper and lower bounds for the explosion time are established, as well as an algorithm to compute the explosion time (under some restrictions). This algorithm is then applied to computing the critical moments, which are shown to be finite for all maturities. The critical moments are related to large and small strike asymptotics of the implied volatility. Joint work with Arpad Pinter and Christoph Gerstenecker.

TBA

Antoine Jacquier (Imperial College London)

March 15, 16:00 – 16:40

Target volatility option pricing in lognormal fractional SABR model

Tai-ho Wang (Baruch College of The City University of New York)

March 15, 16:40 – 17:20

We examine the pricing of target volatility options in the lognormal fractional SABR model. A decomposition formula by Ito's calculus yields a theoretical replicating strategy for the target volatility option, assuming the accessibility of all variance swaps and swaptions. The same formula also suggests an approximation formula for the price of target volatility option in small time by the technique of freezing the coefficient. Alternatively, we also derive closed formed expressions for a small volatility of volatility expansion of the price of target volatility option. Numerical experiments show accuracy of the approximations in a reasonably wide range of parameters. The talk is based on a joint work with Elisa Alos, Rupak Chatterjee, and Sebastian Tudor.

BSDE and Reflected BSDE driven by G-Brownian Motion

Shige Peng (Shandong University)

March 16, 09:40 – 10:20

Implicit Euler-Maruyama scheme for non-colliding particle systems

Dai Taguchi (Osaka University)

March 16, 10:20 – 11:00

In this talk, we consider a discrete approximation for non-colliding particle systems such as Dyson's Brownian motions with drift and Brownian particle systems with nearest neighbour repulsion. As a numerical analysis of a solution to stochastic differential equation, one often approximates it by using the "explicit" Euler-Maruyama scheme. However, unfortunately, the explicit scheme does not preserve the non-colliding property of non-colliding particle systems. Therefore, we introduce implicit Euler-Maruyama scheme which preserve the non-colliding property, and study its rate of convergence in L^p-norm.

This talk is based on joint work with Hoang-Long Ngo (Hanoi National University of Education).

Malliavin calculus for Dyson Brownian motions

Nobuaki Naganuma (Osaka University)

March 16, 11:20 – 12:00

In this talk, we show existence and continuity of density functions of non-colliding particle systems such as Dyson Brownian motions with a smooth drift and hyperbolic particle systems, by using Malliavin calculus. In the proof, we establish a criterion for existence and continuity of density functions of non-colliding particle systems by using the notion of local non-degeneracy of Wiener functionals introduced by Florit and Nualart (1995) and Naganuma (2013). Since the criterion contains a condition about inverse moments of distance of two particles of the system, we give a class of non-colliding particle systems satisfying the inverse moment conditions. We use the Girsanov transformation to check the inverse moment conditions.

This talk is based on a joint work with Dai Taguchi (Osaka University).

Rough Volatility and No Arbitrage

Mathieu Rosenbaum (Ecole Polytechnique)

March 16, 14:00 – 14:40

Functional central limit theorems for non-symmetric random walks on nilpotent covering graphs

Hiroshi Kawabi (Okayama University)

March 16, 14:40 – 15:20

The long time asymptotics for random walks on infinite graphs is a principal topic for both geometers and probabilistic. A covering graph of a finite graph with a nilpotent covering transformation group is called a nilpotent covering graph, regarded as a generalization of a crystal lattice or the Cayley graph of a finite generated group of polynomial growth.

In this talk, we discuss non-symmetric random walks on nilpotent covering graphs from a view point of the theory of discrete geometric analysis developed by Kotani and Sunada, and give functional central limit theorems for them. We also mention a relationship between the limiting diffusions and distorted Brownian rough paths (discussed in e.g., Friz-Oberhauser (09), Bayer-Friz (13), Friz-Gassiat-Lyons (15), Chevyrev (17+) and Lopusanschi-Simon (17+)).

This talk is based on joint work with Satoshi Ishiwata (Yamagata University) and Ryuya Namba (Okayama University).

Unbiased simulation methods of order two

Arturo Kohatsu-Higa (Ritsumeikan University)

March 16, 16:00 – 16:40

We will present various probabilistic representations based on the parametrix method which may be used for Monte Carlo simulations. In particular, we concentrate on a second order method. This is joint work with P. Andersson (Uppsala) and T. Yuasa (Ritsumeikan Univ.) This research is supported by KAKENHI 24340022, 16H03642, 16K05215

	13 (Tuesday)	14 (Wednesday)	15 (Thursday)	16 (Friday)
09:30 – 09:40	Opening			
09:40 – 10:20	Terry Lyons	Shigeo Kusuoka	Jim Gatheral	Shige Peng
10:20 – 11:00	Yu Ito	Yuji Shinozaki	Paul Gassiat	Dai Taguchi
Coffee Break				
11:20 – 12:00	Yuzuru Inahama	Toshihiro Yamada	Elisa Alòs	Nobuaki Naganuma
Lunch				
14:00 – 14:40	Shigeki Aida	Discussions	Peter Friz	Mathieu Rosenbaum
14:40 – 15:20	Ismael Bailleul		Stefan Gerhold	Hiroshi Kawabi
Coffee Break			Coffee Break	
16:00 – 16:40	David Prömel		Antoine Jacquier	Arturo Kohatsu-Higa
16:40 – 17:20	Thomas Cass		Tai-ho Wang	Closing
	Reception		Dinner	