A QUANTITATIVE SECOND ORDER MINIMALITY CRITERION FOR CAVITIES IN ELASTIC BODIES

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Abstract

In this talk we consider a variational model which describes the competition between elastic and surface energy that has been introduced to study surface instability in morphological evolution of cavities in stressed solids. Consider a cavity in an elastic solid, identified with a smooth compact set $F \subset \mathbb{R}^2$, starshaped with respect to the origin and let u be the elastic displacement. Since the effects of the surface energy are negligible far from the void, we may assume that $u = u_0$ outside a large ball B_{R_0}. The bulk energy then takes the form

$$\int_{B_{R_0} \setminus F} Q(E(u))\, dz,$$

where $E(u)$ is the symmetric gradient and Q is a bilinear form depending on the material. The surface energy is simply assumed to be the length of the boundary of F. Then the energy for a smooth configuration is

$$\mathcal{F}(F,u) := \int_{B_0 \setminus F} Q(E(u))\, dz + \mathcal{H}^1(\partial F)\,.$$

The equilibria are identified with minimizers with respect to variations that keep the volume of the cavity constant.

Aim of the talk will be to explain how to perform a second order variational analysis in order to show that regular critical configurations with positive definite second variation are indeed local minimizer for the functional $\mathcal{F}$.

More precisely, given a smooth configuration (F,u) and a one parameter family of smooth perturbations (F_t, u_t), the second variation is defined as

$$\partial^2 \mathcal{F}(F,u) := \frac{d^2}{dt^2} \mathcal{F}(F_t, u_t)\big|_{t=0}\,.$$

The main result roughly reads as follows: if (F,u) is a smooth critical configuration and $\partial^2 \mathcal{F}(F,u) > 0$, then (F,u) is a strict local minimizer of $\mathcal{F}$. Moreover, we provide a quantitative estimate of the distance from the minimum (F,u) of a given configuration (G,v) sufficiently close to (F,u) proving that

$$\mathcal{F}(G,v) \geq \mathcal{F}(F,u) + c_0 |G \Delta F|^2$$

for some constant $c_0 > 0$.