A QUANTITATIVE SECOND ORDER MINIMALITY CRITERION FOR CAVITIES IN ELASTIC BODIES

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Abstract

In this talk we consider a variational model which describes the competition between elastic and surface energy that has been introduced to study surface instability in morphological evolution of cavities in stressed solids. Consider a cavity in an elastic solid, identified with a smooth compact set $F \subset \mathbb{R}^2$, starshaped with respect to the origin and let $u$ be the elastic displacement. Since the effects of the surface energy are negligible far from the void, we may assume that $u = u_0$ outside a large ball $B_{R_0}$. The bulk energy then takes the form

$$\int_{B_{R_0} \setminus F} Q(E(u)) \, dz,$$

where $E(u)$ is the symmetric gradient and $Q$ is a bilinear form depending on the material. The surface energy is simply assumed to be the length of the boundary of $F$. Then the energy for a smooth configuration is

$$\mathcal{F}(F, u) := \int_{B_{R_0} \setminus F} Q(E(u)) \, dz + \mathcal{H}^1(\partial F).$$

The equilibria are identified with minimizers with respect to variations that keep the volume of the cavity constant.

Aim of the talk will be to explain how to perform a second order variational analysis in order to show that regular critical configurations with positive definite second variation are indeed local minimizer for the functional $\mathcal{F}$.

More precisely, given a smooth configuration $(F, u)$ and a one parameter family of smooth perturbations $(F_t, u_t)$, the second variation is defined as

$$\partial^2 \mathcal{F}(F, u) := \frac{d^2}{dt^2} \mathcal{F}(F_t, u_t) \big|_{t=0}.$$

The main result roughly reads as follows: if $(F, u)$ is a smooth critical configuration and $\partial^2 \mathcal{F}(F, u) > 0$, then $(F, u)$ is a strict local minimizer of $\mathcal{F}$. Moreover, we provide a quantitative estimate of the distance from the minimum $(F, u)$ of a given configuration $(G, v)$ sufficiently close to $(F, u)$ proving that

$$\mathcal{F}(G, v) \geq \mathcal{F}(F, u) + c_0 |G \Delta F|^2$$

for some constant $c_0 > 0$. 

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