Title:
The Neumann eigenvalue problem for the Hermite equation

Abstract:
We will show sharp upper and lower bounds for $\mu_1(\Omega)$, the first nontrivial eigenvalue of the problem

$$\begin{cases} -\Delta u + x \cdot \nabla u = \mu u & \text{in} \quad \Omega \\ \\ \dfrac{\partial u}{\partial \nu} = 0 & \text{on} \quad \partial\Omega, \end{cases}$$

where Ω is a smooth and possibly unbounded domain of $\mathbb{R}^n$ and ν stands for the outward normal to $\partial\Omega$.

We firstly prove that among all sets of $\mathbb{R}^n$ symmetric about the origin, having prescribed Gaussian measure, $\mu_1(\Omega)$ is maximum if and only if Ω is the euclidean ball centered at the origin.

On the other hand we will show that $\mu_1(\Omega) \geq 1$, for any convex domain Ω. Note that the last inequality, that clearly does not depend on the measure of Ω, reduces to an equality if Ω is any n-dimensional strip.

References

[1] B. Brandolini -F. Chiacchio - C. Trombetti, A sharp lower bound for some Neumann eigenvalues of the Hermite operator, preprint (2012), arXiv:1209.6275.

[2] B. Brandolini - F. Chiacchio - A. Henrot - C. Trombetti, An optimal Poincaré-Wirtinger inequality in Gauss space, preprint (2012), arXiv:1209.6469.

[3] F. Chiacchio - G. Di Blasio, Isoperimetric estimates for the first Neumann eigenvalue of Hermite differential equations, Ann. Inst. H. Poincaré Anal. Non Linéaire. Volume 29, Issue 2, 199-216, (2012).