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Abstract

This paper investigates firm values and investment strategies (investment, coupon, and default timing) when several firms make strategic real investments with debt financing. We derive and compare the equilibrium investment strategies in three types of duopoly: (i) two symmetric firms, both of which can issue debt, (ii) two symmetric firms, only one of which (the leader) can issue debt, and (iii) a levered firm versus an unlevered firm. We show that in (iii) and in equilibrium, the levered firm always invests prior to the unlevered firm. Further, we derive the equilibrium in the competitive situation of n levered firms, and show that social loss increases as the number of the firms, n, becomes larger.

Keywords: Finance, investment game, strategic real options, debt financing, capital structure.

1 Introduction

The real options approach has become an increasingly standard framework for the investment timing decision in corporate finance (see Dixit and Pindyck (1994)). Although the early literature on real options investigated the monopolist's investment decisions, recent studies have investigated the problem of several firms competing in the same market from a game theoretic approach (see Boyer, Gravel, and Lasserre (2004) for an overview). In particular, many studies, such as Grenadier (1996), Huisman (2001), and Weeds (2002), analyze a duopoly investment game by incorporating equilibrium into a timing game with

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a real options approach. Other studies have concerned incomplete information between firms (e.g., Lambrecht and Perraudin (2003) and Nishihara and Fukushima (2008)) and agency conflicts in a single firm (e.g., Grenadier and Wang (2005) and Nishihara and Shibata (2008)).

From a different perspective, one of the most important problems in corporate finance is the derivation of the optimal capital structure. The theory of an optimal capital structure obtained through the trade-off between tax advantages and default costs, proposed by Modigliani and Miller (1958) in the 1950s, has subsequently been developed in more recent work, including Leland (1994) and Goldstein, Ju, and Leland (2001). Naturally, studies on capital structure and financing have a deep connection with those concerning the investment timing decision. However, few real options approaches focus on these matters. Noteworthy work in this area includes Mauer and Sarkar (2005), Sundaresan and Wang (2007a), and Sundaresan and Wang (2007b). This body of work simultaneously investigates firm value, investment timing, debt financing, and endogenous bankruptcy in a model where a firm undertakes real investment alongside the issuance of debt.

However, existing literature, like Mauer and Sarkar (2005), Sundaresan and Wang (2007a), and Sundaresan and Wang (2007b), only considers a monopoly and does not reflect on the competitive situation existing between several firms. In this paper, we extend the analysis of Sundaresan and Wang (2007a) to the case where several firms attempt to preempt the market. We derive the equilibrium investment strategies in a timing game between firms that can issue debt. Through this, we clarify the effects of competition upon firm value, investment timing, debt financing, and default timing. In order to analytically derive the equilibrium, we consider the simple situation where more than one firm are not permitted to receive profit flow from the market at the same time [1].

We reveal the effects of debt financing on strategic investment by deriving the equilibrium for the following three types of duopoly:
(i) Competition between two symmetric firms. Both firms can issue debt. This may be interpreted as the situation where each firm has its own lender.
(ii) Competition between two symmetric firms. Only the firm that makes an investment first (the leader) can issue debt, while the other firm (the follower) cannot issue debt. This may be interpreted as the situation where there is only one lender for the investment project.

[1]This assumption is essentially the same as that of Lambrecht and Perraudin (2003) and Weeds (2002)

(iii) Competition between two asymmetric firms. Here one firm can issue debt, while the other is unlevered for exogenous reasons, including a shortage of credit.

Note that in the preemptive equilibrium with competition between unlevered firms, investment takes place at the zero net present value (NPV) point (i.e., when the NPV of the investment is zero). In contrast, we show that in the equilibrium of the duopoly cases (i), (ii), and (iii), investment takes place later than zero-NPV timing and the firm value becomes positive. This results from the possibility of the leader's bankruptcy. In addition, the debt coupons the leader issues become smaller than those of the monopolist, while the firms' leverage and credit spread are unchanged.

In particular, we show that in (iii) the levered firm always wins the race. That is, the levered firm invests with debt financing prior to the unlevered firm, and obtains much larger profit than the unlevered firm. We also observe that the inequality (ii) < (i) < (iii) holds with respect to both the timing of investment and the value of the levered firm.

In addition, we derive the equilibrium strategies in the competitive situation of n symmetric levered firms. As the number of the firms, n, becomes larger, investment take places earlier and the coupons and firm value become smaller. On letting $n \to +\infty$ investment timing hastens to the zero-NPV point and the firm value decreases to 0. Furthermore, we investigate the social loss from the preemptive competition among firms by comparing the outcomes in the preemptive and leader–follower games. We show that the larger the number of firms, the greater the social loss.

The paper is organized as follows. Section 2 introduces the benchmark firm values and the investment strategies of the levered and unlevered monopolists. In Section 3, we derive the firm values and the investment strategies in equilibrium for the three types of duopoly (i), (ii), and (iii). In Section 4, we derive the equilibrium in oligopoly and then investigate the social loss arising from the preemptive competition between firms. Section 5 provides several numerical examples and Section 6 concludes.

2 Monopoly

2.1 Unlevered firm

First, let us explain the setup. This paper follows the one-growth option model in Sundaresan and Wang (2007a).[2] Assume that the firm is risk-neutral and behaves in the interests of equityholders. [3] The firm with no initial assets has an option to enter a new market. The firm can choose the investment time by observing market demand $X(t)$ at time t. The firm collects a profit flow $QX(t)$ by paying a sunk cost I, where $Q(>0)$ and $I(>0)$ are constants. We assume that the firm faces a constant tax rate $\tau \in (0,1)$. For simplicity, we assume that $X(t)$ obeys the following geometric Brownian motion:

$$\mathrm{d}X(t) = \mu X(t)\mathrm{d}t + \sigma X(t)\mathrm{d}B(t), X(0) = x(>0), \tag{1}$$

where μ and $\sigma(>0)$ are constants and $B(t)$ represents the one-dimensional standard Brownian motion. The initial value $X(0) = x$ is a sufficiently small constant so that the firm has to wait for its entry condition to be met.

We now consider the unlevered firm with all-equity financing. The unlevered firm determines its investment time T by solving the following optimal stopping time problem:

$$V_{ae}(x) = \sup_{T\in\mathcal{T}} \mathbb{E}[\int_T^{+\infty} \mathrm{e}^{-rt}(1-\tau)QX(t)\mathrm{d}t - \mathrm{e}^{-rT}I], \tag{2}$$

where $\mathcal{T}$ is a set of all $\mathcal{F}_t$ stopping times ($\mathcal{F}_t$ is the usual filtration generated by $B(t)$) and r denotes the risk-free interest rate satisfying $r > \mu$. Problem (2) is reduced to

$$V_{ae}(x) = \sup_{T\in\mathcal{T}} \mathbb{E}[\mathrm{e}^{-rT}(\Pi(X(T)) - I)],$$

where the function $\Pi(X(T))$ is defined by

$$\Pi(X(T)) = \frac{1-\tau}{r-\mu}QX(T). \tag{3}$$

Then the optimal investment time T^i_{ae} and the firm value $V_{ae}(x)$ are easily calculated as

$$T^i_{ae} = \inf\{t > 0 \mid X(t) \geq x^i_{ae}\}, \tag{4}$$

[2]Sundaresan and Wang (2007a) considers a firm with two sequentially ordered growth options in order to investigate debt overhang.

[3]Throughout this analysis, we use the terminology "equityholders" following Sundaresan and Wang (2007a). The model does not distinguish between equityholders and entrepreneur. Hence, for the remainder of the paper, we can replace equityholders and equity value with entrepreneur and entrepreneurial value, respectively. As another way of looking at this, we may consider that in the unlevered setting the entrepreneur does not issue equity, but has the money necessary for the investment project.

and

$$V_{ae}(x) = \left(\frac{x}{x^i_{ae}}\right)^{\beta} (\Pi(x^i_{ae}) - I). \tag{5}$$

(See, for example, Dixit and Pindyck (1994) and McDonald and Siegel (1986)). Here, β is a positive characteristic root defined by

$$\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}\ (> 1),$$

and the investment trigger x^i_{ae} is

$$x^i_{ae} = \frac{\beta}{\beta - 1}\frac{I}{\Pi(1)}. \tag{6}$$

As is well-known, the investment trigger x^i_{ae} is larger than the zero-NPV trigger $x_{NPV} = I/\Pi(1)$.

2.2 Levered firm

This subsection summarizes the results of the one-growth option case in Sundaresan and Wang (2007a). Consider the levered firm that can issue debt with infinite maturity at investment. In the usual manner, we solve the problem backwards.

Assume that the firm has already invested at time s with market demand $X(s)$ along with issuing debt with coupon c. The equityholders (entrepreneur) have an incentive to default after the debt is in place. They choose the default time T^d so as to maximize the equity value as follows:

$$\begin{aligned} &E(X(s), c) \\ &= \sup_{\substack{T \in \mathcal{T} \\ T \geq s}} \mathbb{E}[\int_s^T \mathrm{e}^{-r(t-s)}(1-\tau)(QX(t) - c)\mathrm{d}t \mid \mathcal{F}_s]. \end{aligned} \tag{7}$$

The optimal default time T^d is $T^d = \inf\{t \geq s \mid X(t) \leq x^d(c)\}$, where the default trigger $x^d(c)$ is the function defined by

$$x^d(c) = \frac{\gamma}{\gamma - 1}\frac{r - \mu}{r}\frac{c}{Q}. \tag{8}$$

Here γ denotes a negative characteristic root defined by

$$\gamma = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}\ (< 0).$$

Then, at time s the equity value $E(X(s), c)$, the debt value $D(X(s), c)$, and the firm value $V(X(s), c) = E(X(s), c) + D(X(s), c)$ are expressed as

$$E(X(s), c) = \Pi(X(s)) - \frac{(1-\tau)c}{r} - \left(\Pi(x^d(c)) - \frac{(1-\tau)c}{r}\right)\left(\frac{X(s)}{x^d(c)}\right)^{\gamma} \tag{9}$$

$$D(X(s), c) = \mathbb{E}[\int_s^{T^d} \mathrm{e}^{-r(t-s)} c \mathrm{d}t + \mathrm{e}^{-r(T^d - s)}(1-\alpha)\Pi(X(T^d)) \mid \mathcal{F}_s] \tag{10}$$

$$= \frac{c}{r} - \left(\frac{c}{r} - (1-\alpha)\Pi(x^d(c))\right)\left(\frac{X(s)}{x^d(c)}\right)^\gamma \tag{11}$$

$$V(X(s), c) = \Pi(X(s)) + \frac{\tau c}{r} - \left(\alpha\Pi(x^d(c)) + \frac{\tau c}{r}\right)\left(\frac{X(s)}{x^d(c)}\right)^\gamma \tag{12}$$

for $X(s) \geq x^d(c)$, where $\alpha(\geq 0)$ is a given constant representing the default cost. Note that the debtholders collect the entire default value, i.e., $(1-\alpha)\Pi(x^d(c))$.

The equityholders (entrepreneur) choose the investment trigger T^i and coupon c to maximize the firm value (12). That is, the problem becomes the following:

$$V_{de}(x) = \sup_{\substack{T \in \mathcal{T} \\ c(\geq 0): \mathcal{F}_T - measurable}} \mathbb{E}[\mathrm{e}^{-rT}(V(X(T), c) - I)]. \tag{13}$$

Problem (13) can be interpreted as follows. Assume that the debtholders lend K for the debt. The equityholders' (entrepreneur's) value at investment time T is then

$$E(X(T), c) + K - I, \tag{14}$$

while the debtholders' value at T becomes

$$D(X(T), c) - K. \tag{15}$$

Since the sum of (14) and (15) is equal to $V(X(T), c) - I$, the solution of problem (13) is optimal for both the equityholders and the debtholders. The amount K determines the asset allocation between equityholders and debtholders, but in this paper we do not consider the allocation problem. [4]

Note that $\arg\max_{c \geq 0} V(X(s), c)$ becomes

$$c(X(s)) = \frac{r}{r-\mu} \frac{\gamma - 1}{\gamma} \frac{QX(s)}{h} \ (> 0), \tag{16}$$

for $X(s) > 0$. Here, h is a constant given by

$$h = \left[1 - \gamma\left(1 - \alpha + \frac{\alpha}{\tau}\right)\right]^{-\frac{1}{\gamma}} > 1.$$

By some calculation we can show

$$V(X(s), c(X(s))) = \psi^{-1}\Pi(X(s)), \tag{17}$$

[4] In Mauer and Sarkar (2005) agency conflicts between equityholders and debtholders occur at the investment time because the amount K is fixed prior to investment. In contrast, such conflicts do not arise in Sundaresan and Wang (2007a) and the current analysis because the price K is adjusted through negotiation at the time of the investment. The difference between problem (13) and the "first-best" scenario in Mauer and Sarkar (2005) is whether the coupon c is controllable.

where the function $\Pi(\cdot)$ is defined by (3) and ψ is a constant given by

$$\psi = \left[1 + \frac{\tau}{(1-\tau)h}\right]^{-1} < 1.$$

As a result, problem (13) can be rewritten as

$$V_{de}(x) = \sup_{T \in \mathcal{T}} \mathbb{E}[\mathrm{e}^{-rT}(\psi^{-1}\Pi(X(s)) - I)].$$

Thus, the optimal investment time of (13) is

$$T^i = \inf\{t > 0 \mid X(t) \geq x^i\},$$

and the optimal coupon is $c(x^i)$, where the investment trigger x^i is defined by

$$x^i = \psi x^i_{ae} < x^i_{ae}. \tag{18}$$

Recall that x^i_{ae} is defined by (6). From (8) and (16), we have a default trigger $x^d(c(x^i)) = x^i/h$. The firm value $V_{de}(x)$ at the initial time becomes

$$V_{de}(x) = \left(\frac{x}{x^i}\right)^{\beta}(\psi^{-1}\Pi(x^i) - I) = \psi^{-\beta}V_{ae}(x). \tag{19}$$

The investment trigger x^i of the levered firm lies between the levered firm's zero-NPV trigger ψx_{NPV} and the unlevered firm's optimal trigger x^i_{ae}. Note that the unlevered firm's problem (2) corresponds to (13) with $c = 0$. Naturally, the levered firm's value (19) is greater than that of the unlevered firm (5). The leverage LV and the credit spread CS at the time of investment are calculated as

$$\begin{aligned} LV &= \frac{D(x^i, c(x^i))}{V(x^i, c(x^i))} \\ &= \frac{\gamma - 1}{\gamma}\frac{\psi(1-\tau)(1-\xi)}{h} \end{aligned} \tag{20}$$

and

$$\begin{aligned} CS &= \frac{c(x^i)}{D(x^i, c(x^i))} - r \\ &= r\frac{\xi}{1-\xi}, \end{aligned} \tag{21}$$

respectively, where ξ is defined by

$$\xi = \left(1 - (1-\alpha)(1-\tau)\frac{\gamma}{\gamma - 1}\right)h^{\gamma}.$$

Note that $0 < \xi < 1$ and both (20) and (21) do not depend on the investment trigger x^i. For further details of the results concerning the levered monopolist, see the one-growth option case in Sundaresan and Wang (2007a).

3 Duopoly

This section considers the competition between two firms with complete information and focuses on the strategic investment with debt financing. Assume that each firm receives a cash flow $Q_2 X(t)$ when both firms are active in the market. In order to show the essence of the firms' preemptive activities, Sections 3.1–3.3 assume $Q_2 = 0$ as in Lambrecht and Perraudin (2003) and Weeds (2002). This means that the market is small enough to be supplied by a single firm. After Section 3.1 describes the benchmark case of the competition between unlevered firms, Sections 3.2 and 3.3 investigate the situation of two symmetric firms that can issue debt, and two asymmetric firms; that is, a levered firm versus an unlevered firm. Section 3.4 gives a brief comment on the general case of $Q_2 \in (0, Q)$ (negative externalities), though we are unable to present the analytical derivation.

3.1 Competition between unlevered firms

This subsection provides the well-known outcome under competition between two unlevered firms (see, for example, Huisman (2001)). Let $L_{ae}(X(s))$ and $F_{ae}(X(s))$ denote the expected discounted value (at time s) of a firm that enters the market first (the leader) at $X(s)$ and that of the other firm that responds optimally to the leader (the follower). It follows from $Q_2 = 0$ that the follower has no opportunity for investment. Accordingly, the follower's and leader's values become $F_{ae}(X(s)) = 0$ and $L_{ae}(X(s)) = \Pi(X(s)) - I$, respectively. In the situation where neither firm has invested, each firm attempts to invest earlier than the other in order to obtain the leader's payoff $L_{ae}(X(s))$ when the leader's payoff $L_{ae}(X(s))$ is larger than the follower's payoff $F_{ae}(X(s))$. Through preemption, each firm tries to invest at the zero-NPV point $X(s) = x_{NPV}$, [5] which is the solution of $\Pi(X(s)) - I = 0$ in equilibrium. Consequently, each firm's value becomes zero. There are no equilibriums other than the above (referred to as the preemptive equilibrium). Note that the outcome remains unchanged in the setting where n unlevered firms compete.

3.2 Competition between two symmetric firms

This section considers two types of competition between two symmetric firms with debt financing. We first consider Duopoly (i), where both firms, regardless of whether they

[5] Following Weeds (2002), this paper assumes that one of the firms is chosen as a leader when the firms try to invest at the same threshold. For details of the timing game, see Appendix A.

invest first, can issue debt.

As usual, we begin where one of the firms (the leader) has already invested at $X(s)$. The leader's firm value, denoted by $L_{de}(X(s))$, is $L_{de}(X(s)) = \psi^{-1}\Pi(X(s)) - I$ because from point s the leader can obtain the monopolist's cash flow $QX(t)$ and choose the monopolist's default strategy since $Q_2 = 0$. Recall that the leader investing at $X(s)$ chooses the optimal coupon (16) and obtains firm value (17).

On the other hand, the other firm's value (the follower's), denoted by $F_{de}(X(s))$, is calculated as follows:

$$\begin{aligned} F_{de}(X(s)) &= \left(\frac{X(s)}{x^d(c(X(s)))}\right)^{\gamma} V_{de}(x^d(c(X(s)))) \\ &= \begin{cases} h^{\gamma-\beta}\left(\dfrac{X(s)}{x^i}\right)^{\beta}(\psi^{-1}\Pi(x^i) - I) & (0 < X(s) < hx^i) \\ h^{\gamma}\left[\psi^{-1}\Pi\left(\dfrac{X(s)}{h}\right) - I\right] & (X(s) \geq hx^i). \end{cases} \end{aligned} \tag{22}$$

Eq. (22) is the value of the option to invest after the leader's bankruptcy. Note that the follower chooses the same investment trigger (of course, *is not at the same time*), coupon, and default trigger as the monopolist, i.e., x^i, $c(x^i)$, and $x^d(c(x^i))$. Unlike $F_{ae}(X(s)) = 0$ in Section 3.1, $F_{de}(X(s)) > 0$ holds for all $X(s) > 0$. As discussed in problem (13), the equityholders (entrepreneur) of each firm choose the investment time and coupon to maximize firm value. Accordingly, the equityholders of each firm attempt to preempt the rival when the leader's incentive is positive, i.e., $L_{de}(X(s)) > F_{de}(X(s))$. We have the following proposition.

Proposition 1 There exists a unique x_P satisfying $L_{de}(x_P) = F_{de}(x_P)$ in the interval $(\psi x_{NPV}, x^i)$. In Duopoly (i) only the following preemptive equilibrium occurs. Each firm attempts to invest at

$$T_L^i = \inf\{t > 0 \mid X(t) \geq x_P\}$$

and one of the firms executes the investment as a leader at time T_L^i, along with issuing debt with coupon $c(x_P)$. The leader then defaults at

$$T_L^d = \inf\{t > T_L^i \mid X(t) \leq x_P/h\}.$$

After the leader's bankruptcy, the other firm, as a follower, invests at

$$T_F^i = \inf\{t > T_L^d \mid X(t) \geq x^i\}$$

along with issuing debt with coupon $c(x^i)$, and then defaults

$$T_F^d = \inf\{t > T_F^i \mid X(t) \leq x^i/h\}.$$

The firm value at the initial time becomes

$$h^{\gamma-\beta}V_{de}(x). \tag{23}$$

Proof The function $\psi^{-1}\Pi(\cdot)$ is linear and the function $F_{de}(\cdot)$ is convex by (22). The function $F_{de}(\cdot) - L_{de}(\cdot)$ is then convex. Hence, from

$$L_{de}(\psi x_{NPV}) = 0 < F_{de}(\psi x_{NPV})$$

and

$$L_{de}(x^i) = V_{de}(x^i) > F_{de}(x^i)$$

there exists a unique x_P satisfying $L_{de}(x_P) = F_{de}(x_P)$ in the interval $(\psi x_{NPV}, x^i)$. The solution x_P is the preemptive trigger which leads to the preemptive equilibrium in the proposition. For details, see Appendix A. Since

$$L_{de}(X(s)) > \frac{L_{de}(X(s)) + F_{de}(X(s))}{2} \quad (X(s) > x_P),$$

the leader's incentive is always positive for $X(s) > x_P$. This implies that a joint investment-type equilibrium does not arise.

In the preemptive equilibrium, the follower's value at the initial time can be calculated as follows:

$$\begin{aligned}
\left(\frac{x}{x_P}\right)^{\beta} F_{de}(x_P) &= \left(\frac{x}{x_P}\right)^{\beta} h^{\gamma-\beta}\left(\frac{x_P}{x^i}\right)^{\beta}(\psi^{-1}\Pi(x^i) - I) \\
&= h^{\gamma-\beta}\left(\frac{x}{x^i}\right)^{\beta}(\psi^{-1}\Pi(x^i) - I) \\
&= h^{\gamma-\beta}V_{de}(x).
\end{aligned}$$

Recall that the leader' value is the same as the follower's by the definition of the preemptive trigger x_P. □

The preemptive trigger x_P may be smaller than the unlevered firm's zero-NPV point x_{NPV}, though of course it is larger than in the levered case, ψx_{NPV}. In fact, and as presented in Section 5, for many practical parameter values we observe $x_P < x_{NPV}$. Proposition 1 shows that the leader has a smaller investment trigger, coupon, and default trigger than the monopolist (or follower), i.e., $x_P < x^i$, $c(x_P) < c(x^i)$ and, $x^d(c(x_P)) = x_P/h < x^d(c(x^i)) = x^i/h$. Both firms' leverage and credit spared at the investment time remain unchanged from those of the monopolist, i.e., (20) and (21), respectively. This is because even with the fear of preemption by the rival the firm can optimize its capital structure. The firm value (23) is $h^{\gamma-\beta}(< 1)$ times the levered monopolist value (19) because of the preemptive competition.

A firm's endogenous default decision generates a positive firm value, in spite of the assumption $Q_2 = 0$. This feature contrasts with earlier results in the extant literature. In Weeds (2002) and Nishihara and Ohyama (2008) a leader does not always obtain a profit from the market because it takes a random development term from investment until the completion of the project. The random development term generates a positive value under competition. In Lambrecht and Perraudin (2003) incomplete information about the rival firm's strategy plays a role in generating a positive value under competition.

Next, let us turn to Duopoly (ii) where only the leader can issue debt. This may be interpreted as the situation where only one lender exists for the investment project. The firm value of the leader who invests at $X(s)$ does not change from $L_{de}(X(s))$. On the other hand, the firm value, denoted by F_{ae}^{de}, of the follower who takes the optimal response is reduced to the following:

$$\begin{aligned} F_{ae}^{de}(X(s)) &= \left(\frac{X(s)}{x^d(c(X(s)))}\right)^{\gamma} V_{ae}(x^d(c(X(s)))) \\ &= \begin{cases} h^{\gamma-\beta}\left(\dfrac{X(s)}{x_{ae}^i}\right)^{\beta}(\Pi(x_{ae}^i) - I) & (0 < X(s) < hx_{ae}^i) \\ h^{\gamma}(\Pi(x_{ae}^i) - I) & (X(s) \geq hx_{ae}^i). \end{cases} \end{aligned} \tag{24}$$

The firms attempt to preempt each other for $X(t)$ satisfying $L_{de}(X(t)) > F_{ae}^{de}(X(t))$. [6] We obtain the following proposition in Duopoly (ii).

Proposition 2 There exists a unique solution of $L_{de}(\hat{x_P}) = F_{ae}^{de}(\hat{x_P})$ in the interval $(\psi x_{NPV}, x^i)$. In Duopoly (ii) only the following preemptive equilibrium occurs. Each firm tries to invest at

$$\hat{T_L^i} = \inf\{t > 0 \mid X(t) \geq \hat{x_P}\}$$

and one of the firms executes the investment as a leader at time $\hat{T_L^i}$ along with issuing debt with coupon $c(\hat{x_P})$. Then the leader defaults at

$$\hat{T_L^d} = \inf\{t > \hat{T_L^i} \mid X(t) \leq \hat{x_P}/h\}.$$

After the leader's bankruptcy, the remaining firm, as a follower, invests at

$$\hat{T_F^i} = \inf\{t > \hat{T_L^d} \mid X(t) \geq x_{ae}^i\}$$

[6] This paper considers a model where the equityholders (entrepreneur) attempt to maximize the firm value as discussed in problem (13). This paper does not consider the debtholders' optimal strategy. As will be noted in Section 6, it remains an important topic for future work to analyze how the allocation between equityholders and debtholders changes with competition among entrepreneurs.

without debt. The firm value at the initial time becomes

$$h^{\gamma-\beta}V_{ae}(x). \tag{25}$$

Proof The proof is given in the same way as for Proposition 1, and is therefore omitted. □

It can be easily checked that $F_{ae}^{de}(X(s)) < F_{de}(X(s))$ for $X(s) > 0$. This implies that the leader has a smaller investment trigger, coupon, and default trigger than the leader in Duopoly (i), i.e., $\hat{x_P} < x_P$, $c(\hat{x_P}) < c(x_P)$, and $x^d(c(\hat{x_P})) = \hat{x_P}/h < x^d(c(x_P)) = x_P/h$. The firms' leverage and credit spread are the same as (20) and (21) in monopoly. The firm value (25) is $V_{de}(x)/V_{ae}(x)(< 1)$ times that of (23) in Duopoly (i). Compared with Duopoly (i), more severe preemptive competition occurs in Duopoly (ii) because the leader enjoys not only a market advantage, but also the advantage of adjusting its capital structure.

3.3 Competition between the levered and unlevered firms

This subsection considers Duopoly (iii): a levered firm versus an unlevered firm that is not allowed to issue debt for some exogenous reason, such as a shortage of credit. The firm value of the levered firm that invests as a leader at $X(s)$ agrees with $L_{de}(X(s))$, while the firm value of the unlevered firm that responds optimally as a follower is equal to $F_{ae}^{de}(X(s))$ given by (24). Conversely, the firm value of the unlevered firm that invests as a leader at $X(s)$ becomes $L_{ae}(X(s))$, while the firm value of the levered firm acting as a follower is $F_{ae}(X(s)) = 0$. [7]

The levered firm has an incentive to preempt the unlevered firm for $X(s)$ satisfying $L_{de}(X(s)) > F_{ae}(X(s)) = 0$, i.e., $X(s) > \psi x_{NPV}$. On the other hand, the unlevered firm attempts to become the leader for $X(s)$ satisfying $L_{ae}(X(s)) > F_{ae}^{de}(X(s))$. Considering this, we have the following proposition in Duopoly (iii).

Proposition 3 There exists a unique solution $\tilde{x_P}$ of $L_{ae}(\tilde{x_P}) = F_{ae}^{de}(\tilde{x_P})$ in the interval (x_{NPV}, x_{ae}^i). The outcomes in Duopoly (iii) are classified into the following two cases.
(a) $\tilde{x_P} < x^i$
Only the following preemption equilibrium occurs. The levered firm invests at

$$\tilde{T_L^i} = \inf\{t > 0 \mid X(t) \geq \tilde{x_P}\}$$

[7] As shown in Proposition 3, the levered firm always becomes a leader in equilibrium, and therefore the order is never realized.

along with issuing debt of coupon $c(\tilde{x_P})$, and then defaults

$$\tilde{T_L^d} = \inf\{t > \tilde{T_L^i} \mid X(t) \leq \tilde{x_P}/h\}.$$

After the levered firm's bankruptcy, the unlevered firm invests at

$$\tilde{T_{Fa}^i} = \inf\{t > \tilde{T_L^d} \mid X(t) \geq x_{ae}^i\}.$$

The levered firm value at the initial time is equal to

$$\left(\frac{x}{\tilde{x_P}}\right)^\beta (\psi^{-1}\Pi(\tilde{x_P}) - I). \tag{26}$$

The unlevered firm value agrees with (25).

(b) $\tilde{x_P} \geq x^i$

Only the following equilibrium (referred to as the dominant leader-type equilibrium) occurs. The levered firm invests at

$$T^i = \inf\{t > 0 \mid X(t) \geq x^i\}$$

along with issuing debt of coupon $c(x^i)$, and then defaults

$$T^d = \inf\{t > T^i \mid X(t) \leq x^i/h\}.$$

After the levered firm's bankruptcy the unlevered firm invests at

$$\tilde{T_{Fb}^i} = \inf\{t > T^d \mid X(t) \geq x_{ae}^i\}.$$

The levered firm value at the initial time is the same as that of the monopolist, $V_{de}(x)$, given by (19). The unlevered firm value is (25).

Proof The function $L_{ae}(\cdot)$ is linear and the function $F_{ae}^{de}(\cdot)$ is convex by (24). Accordingly, the function $F_{ae}^{de}(\cdot) - L_{ae}^{de}(\cdot)$ is convex. Then, from

$$L_{ae}(x_{NPV}) = 0 < F_{ae}^{de}(x_{NPV})$$

and

$$L_{ae}(x_{ae}^i) = V_{ae}(x_{ae}^i) > F_{ae}^{de}(x_{ae}^i)$$

there exists a unique solution $\tilde{x_P}$ of $L_{ae}^{de}(\tilde{x_P}) = F_{ae}^{de}(\tilde{x_P})$ in the interval $(\psi x_{NPV}, x_{ae}^i)$. Since the unlevered firm's preemptive region $(\tilde{x_P}, +\infty)$ is included in the levered firm's preemptive region $(\psi x_{NPV}, +\infty)$, in equilibrium the levered firm always invests first.

First, assume (a) $\tilde{x_P} < x^i$. In equilibrium, and because of preemption, the levered firm invests at the preemptive trigger $\tilde{x_P}$ and the unlevered firm takes the follower's best response, i.e., $T^{\tilde{i}}_{Fa}$. See Appendix A. The levered and unlevered firms' values at the initial time can be readily derived as (26) and (25), respectively. By

$$L_{de}(X(s)) > \frac{L_{de}(X(s)) + F_{ae}(X(s))}{2} \quad (X(s) > \psi x_{NPV})$$

we can show that no joint investment-type equilibrium exists.

Second, we consider the case (b) $\tilde{x_P} \geq x^i$. Since the unlevered firm does not preempt the levered firm at $X(s) = x^i$, the levered firm can take the best investment strategy. On the other hand, the unlevered firm invests at the follower's optimal timing $T^{\tilde{i}}_{Fb}$. See Appendix A. The levered and unlevered firms' values at the initial time can be easily calculated. We can also easily show that no joint investment-type equilibrium occurs. □

As explained in Huisman (2001) and Kong and Kwok (2007), three types of equilibrium may arise, namely preemptive, dominant leader-type, and joint investment-type. In (a) in Proposition 3 the preemptive equilibrium occurs, while the dominant leader-type equilibrium occurs in (b). In both cases, the levered firm that enjoys the optimal capital structure becomes the leader. The result is realistically intuitive. For quite a large τ, which leads a small x^i, condition (b) is satisfied. In (b) the levered firm is dominant owing to its substantial tax advantage over the unlevered firm.

Let us look at the investment strategies in Proposition 3. Note that the unlevered firm's investment trigger is the same as the unlevered monopolist's. With respect to the levered firm's strategy in (a), we can show inequalities $\hat{x_P} < \tilde{x_P} < x^i$, $c(\hat{x_P}) < c(\tilde{x_P}) < c(x^i)$, and $x^d(c(\hat{x_P})) = \hat{x_P}/h < x^d(c(\tilde{x_P})) = \tilde{x_P}/h < x^d(c(x^i)) = x^i/h$. As in the previous propositions, the firm's leverage and credit spread at investment time are unchanged from the (20) and (21) of the monopolist. Note that the trigger $\tilde{x_P}$, unlike x_P, is always larger than the unlevered firm's zero-NPV trigger x_{NPV}. The inequality $\tilde{x_P} > x_P$ holds for most parameter values, as shown in Section 5, though it cannot be theoretically proven. In (b), the levered firm can take the best strategy. i.e., the monopolist's strategy, because of the strong tax effect.

We now consider the firm value in Proposition 3. In both cases, the unlevered firm must wait for the leader's bankruptcy. Due to the waiting time, the unlevered firm's value (25) becomes $h^{\gamma-\beta}(< 1)$ times the monopolist's value. Note that the unlevered firm's value is the same in both cases, in spite of $T^{\tilde{i}}_{Fa} \neq T^{\tilde{i}}_{Fb}$. The levered firm value in (a) is also reduced from that of the monopolist because of the suboptimal investment timing. By $\tilde{x_P} > x_P > \hat{x_P}$, the levered firm's value (26) becomes larger than (25) and (23) in Duopolies (i) and (ii). The levered firm's value in (b) also agrees with that of the monopolist because it can take up the optimal strategy. To sum up, the fact that the rival changes from a levered to unlevered firm means a decline in the rival's competitive power and therefore an increase in the levered firm's value. Note that in both cases the levered firm's value exceeds the (25) of the unlevered firm.

3.4 Case of $Q_2 > 0$

This subsection provides a brief explanation of the results in the general case such that $0 < Q_2 < Q$, although the setting does not allow us to show clear results. As a benchmark, we consider competition between two unlevered firms. By $Q_2 > 0$, the follower can enter the market where the leader survives when the market demand $X(s)$ is sufficiently large. The leader's profit is reduced from $QX(t)$ to $Q_2X(t)$ after the follower's entry. Since the leader's incentive is smaller than in the case of $Q_2 = 0$, the preemption trigger becomes larger than the zero-NPV trigger x_{NPV}. This generates a positive firm value in equilibrium in the case of $Q_2 \in (0, Q)$.

We now consider Duopolies (i)–(iii). In every case, the follower may invest for large $X(s)$ prior to the leader's default. Note that the follower in (ii) and (iii) never defaults. This changes the leader's default trigger in the market where both are active from $x^d(c)$ to $x^d(c)Q/Q_2$. Thus, in (ii) and (iii) both the equity and debt values of the leader are reduced from (9) and (11). Expecting the possibility of the follower's interception, the leader issues debt with a smaller coupon than $c(X(s))$. On the other hand, because of the decrease in the leader's value and the increase in the follower's value, the preemption triggers, denoted by $\hat{x_P}'$ and $\tilde{x_P}'$ in (ii) and (iii) with $Q_2 \in (0, Q)$, become larger than $\hat{x_P}$ and $\tilde{x_P}$, respectively. With the trade-off between these two effects, it is ambiguous whether the leader's coupon in the investment time in (ii) and (iii) with $Q_2 \in (0, Q)$ is smaller than $c(\hat{x_P})$ and $c(\tilde{x_P})$. The leverage and credit spread may also change from those of the monopolist.

In Duopoly (i), the analysis is more complicated. The follower can choose its coupon taking account of the outcome of the exit-timing game discussed in Murto (2004) when it enters the market where the leader survives. The follower is likely to choose a smaller coupon than the leader so that it can win the exit-timing game (i.e., collect a monopolistic profit flow $QX(t)$ following the leader's bankruptcy). In this case, the leader's default trigger changes from $x^d(c)$ to $x^d(c)Q/Q_2$, which implies a similar outcome to (ii) and (iii) with $Q_2 \in (0, Q)$. The inequality $\hat{x_P}' < x'_P, \tilde{x_P}'$ is unchanged, where x'_P denotes the preemption trigger in (ii) with $Q_2 \in (0, Q)$.

4 Oligopoly

4.1 Competition among n levered firms

Throughout this section, we assume that the market is small enough for demand to be met by a single firm, i.e., $Q_2 = 0$. In this section, we generalize Duopoly (i) to the situation of n firms that can issue debt. We obtain the following proposition.

Proposition 4 Under competition among n firms, only the following preemptive equilibrium occurs.

Each firm tries to invest at

$$T^i_{(n)} = \inf\{t > 0 \mid X(t) \geq x^i_{(n)}\}$$

and one of the firms (denoted by Firm n) executes the investment at $T^i_{(n)}$ [8] along with issuing debt with coupon $c(x^i_{(n)})$. Then Firm n defaults at

$$T^d_{(n)} = \inf\{t > T^i_{(n)} \mid X(t) \leq x^i_{(n)}/h\}.$$

After Firm n's default, the remaining ($n-1$ firms) attempt to invest at

$$T^i_{(n-1)} = \inf\{t > T^d_{(n)} \mid X(t) \geq x^i_{(n-1)}\}$$

and one of the firms (denoted by Firm $n-1$) executes the investment at $T^i_{(n-1)}$ along with issuing debt with coupon $c(x^i_{(n-1)})$. Then Firm $n-1$ defaults at

$$T^d_{(n-1)} = \inf\{t > T^i_{(n-1)} \mid X(t) \leq x^i_{(n-1)}/h\}.$$

$\vdots$

After Firm 2's default, the last firm (Firm 1) invests at

$$T^i_{(1)} = \inf\{t > T^d_{(2)} \mid X(t) \geq x^i_{(1)}\}$$

along with issuing debt with coupon $c(x^i_{(1)})$, and then defaults at

$$T^d_{(1)} = \inf\{t > T^i_{(1)} \mid X(t) \leq x^i_{(1)}/h\}.$$

Here the investment trigger $x^i_{(k)}$ of Firm k is defined by the unique solution of

$$\psi^{-1}\Pi(x^i_{(k)}) - I = h^{(k-1)(\gamma-\beta)} \left(\frac{x^i_{(k)}}{x^i}\right)^{\beta} (\psi^{-1}\Pi(x^i) - I) \quad (\psi x_{NPV} < x^i_{(k)} \leq x^i). \qquad (27)$$

[8] As in Proposition 1, we assume that one of the firms is chosen with a probability of $1/n$. See Appendix A.

The investment triggers $x^i_{(k)}$ satisfy

$$\psi x_{NPV} < x^i_{(n)} < x^i_{(n-1)} < \ldots < x^i_{(2)} = x_P < x^i_{(1)} = x^i. \tag{28}$$

In equilibrium, the firm value at the initial time is equal to

$$h^{(n-1)(\gamma-\beta)} V_{de}(x). \tag{29}$$

As $n \to +\infty$, the firm value (29) and the preemption trigger $x^i_{(n)}$ converge to 0 and ψx_{NPV}, respectively.

Proof We first prove that Eq. (27) has a unique solution in the interval. It is explicit that (27) has a unique solution $x^i_{(1)} = x^i$ for $k = 1$. Consider $k \geq 2$. Note that the left-hand side of (27) is linear, and the right is convex, with respect to $x^i_{(k)}$ for the interval. Then, from

$$\psi^{-1}\Pi(\psi x_{NPV}) - I = 0 < h^{(k-1)(\gamma-\beta)} \left(\frac{\psi x_{NPV}}{x^i}\right)^{\beta} (\psi^{-1}\Pi(\psi x_{NPV}) - I) \tag{30}$$

and

$$\psi^{-1}\Pi(x^i) - I > h^{(k-1)(\gamma-\beta)}(\psi^{-1}\Pi(x^i) - I), \tag{31}$$

there exists a unique solution $x^i_{(k)} \in (\psi x_{NPV}, x^i)$. The inequality (28) follows from the feature that $h^{(k-1)(\gamma-\beta)}$ in (27) monotonically decreases with k.

We now consider the firm value (at the initial time) of Firm k in Proposition 4. The value can be calculated as

$$\begin{aligned}
& \left(\frac{x}{x^i_{(n)}}\right)^{\beta} h^{\gamma} \left(\frac{x^i_{(n)}}{h x^i_{(n-1)}}\right)^{\beta} h^{\gamma} \cdots \left(\frac{x^i_{(k+2)}}{h x^i_{(k+1)}}\right)^{\beta} h^{\gamma} \left(\frac{x^i_{(k+1)}}{h x^i_{(k)}}\right)^{\beta} (\psi^{-1}\Pi(x^i_{(k)}) - I) \\
= \; & h^{(n-k)(\gamma-\beta)} \left(\frac{x}{x^i_{(k)}}\right)^{\beta} (\psi^{-1}\Pi(x^i_{(k)}) - I) \\
= \; & h^{(n-k)(\gamma-\beta)} \left(\frac{x}{x^i_{(k)}}\right)^{\beta} h^{(k-1)(\gamma-\beta)} \left(\frac{x^i_{(k)}}{x^i}\right)^{\beta} (\psi^{-1}\Pi(x^i) - I) && (32) \\
= \; & h^{(n-1)(\gamma-\beta)} V_{de}(x), && (33)
\end{aligned}$$

where (32) results from the definition (27). The value (33) does not depend on k—in other words, the firms are indifferent to the order of investment. This implies that the strategy stated in Proposition 4 is the equilibrium strategy (for details, refer Appendix A). It can be easily shown that no joint investment-type equilibrium exists by the same discussion as in the proof of Proposition 1. The firm value in the equilibrium is equal to (33)=(29). As $n \to +\infty$, (29) $\downarrow 0$. Particularly, Firm n's value is

$$\left(\frac{x}{x^i_{(n)}}\right)^{\beta} (\psi^{-1}\Pi(x^i_{(n)}) - I) \downarrow 0 \quad (n \to +\infty),$$

which implies $x^i_{(n)} \downarrow \psi x_{NPV}$ $(n \to +\infty)$. $\square$

From Proposition 4 we have the inequalities $x^i_{(k+1)} < x^i_{(k)}$, $c(x^i_{(k+1)}) < c(x^i_{(k)})$, and $x^d(c(x^i_{(k+1)})) = x^i_{(k+1)}/h < x^d(c(x^i_{(k)})) = x^i_{(k)}/h$ (see Table 1). As in the previous propositions, the leverage and credit spread at the investment time are unchanged from those of the monopolist. The firm value (29) is $h^{(\gamma-\beta)(n-1)}(< 1)$ times the monopolist's value $V_{de}(x)$. The firm value monotonically decreases to 0 as the number of firms, n, increases. This can be interpreted as the case where a positive excess profit that arises in oligopoly (i.e., finite n) vanishes in the competitive market (i.e., infinite n). In the competitive market where an infinite number of firms compete, every firm attempts to invest at the zero-NPV trigger ψx_{NPV}. Our results in the limiting case are similar to the results in Lambrecht and Perraudin (2003).

4.2 Social loss due to preemption

This subsection focuses on social loss from preemptive competition among firms. We first consider the outcome of the leader–follower game where the order of the firms is exogenously given in advance. Without fear of preemption by the other firms, every firm chooses the monopolist's strategy, i.e., investment trigger x^i, coupon $c(x^i)$, and default trigger $x^d(c(x^i)) = x^i/h$ (see Table 2). The firm value of Firm k, which invests after $n-k$ firms default, is derived as

$$h^{(n-k)(\gamma-\beta)}V_{de}(x). \tag{34}$$

By comparing Table 1 with Table 2, we can see the inefficiency caused by the preemption. We can see that the value of Firm 1, which is given the worst role in the leader–follower game, in Table 2 agrees with the value of all firms in the preemptive equilibrium in Table 1. The total sum of the values of n firms is

$$nh^{(\gamma-\beta)(n-1)}V_{de}(x) \downarrow 0 \quad (n \to +\infty) \tag{35}$$

in the preemptive equilibrium, while the sum in the leader–follower game is

$$\sum_{k=1}^{n} h^{(\gamma-\beta)(k-1)}V_{de}(x) = \frac{1-h^{(\gamma-\beta)n}}{1-h^{\gamma-\beta}}V_{de}(x) \uparrow \frac{V_{de}(x)}{1-h^{\gamma-\beta}} \quad (n \to +\infty). \tag{36}$$

We define the (relative) social loss from preemption by n firms, denoted by $Loss(n)$, as $Loss(n) = 1-$ (35)/(36). Then we have

$$Loss(n) = 1 - \frac{nh^{(\gamma-\beta)(n-1)}(1-h^{\gamma-\beta})}{1-h^{(\gamma-\beta)n}} \uparrow 1 \quad (n \to +\infty). \tag{37}$$

From (37) we can state that an increase in the number of firms, n, causes severe preemptive competition and an inefficient outcome with greater social loss.

Table 1: Preemptive game.

	Firm n	Firm $n-1$	$\cdots$	Firm 2	Firm 1
Investment	$x^i_{(n)} <$	$x^i_{(n-1)} <$	$\cdots <$	$x^i_{(2)} = x_P <$	$x^i_{(1)} = x^i$
Coupon	$c(x^i_{(n)}) <$	$c(x^i_{(n-1)}) <$	$\cdots <$	$c(x^i_{(2)}) <$	$c(x^i_{(1)})$
Default	$x^i_{(n)}/h <$	$x^i_{(n-1)}/h <$	$\cdots <$	$x^i_{(2)}/h <$	$x^i_{(1)}/h$
Value	$h^{(n-1)(\gamma-\beta)}V_{de}(x)$	$h^{(n-1)(\gamma-\beta)}V_{de}(x)$	$\cdots$	$h^{(n-1)(\gamma-\beta)}V_{de}(x)$	$h^{(n-1)(\gamma-\beta)}V_{de}(x)$

Table 2: Leader-follower game.

	Firm n	Firm $n-1$	$\cdots$	Firm 2	Firm 1
Investment	x^i	x^i	$\cdots$	x^i	x^i
Coupon	$c(x^i)$	$c(x^i)$	$\cdots$	$c(x^i)$	$c(x^i)$
Default	x^i/h	x^i/h	$\cdots$	x^i/h	x^i/h
Value	$V_{de}(x) >$	$h^{\gamma-\beta}V_{de}(x) >$	$\cdots >$	$h^{(n-2)(\gamma-\beta)}V_{de}(x) >$	$h^{(n-1)(\gamma-\beta)}V_{de}(x)$

5 Numerical examples

This section describes the economic implications, using some numerical examples of the theoretical results given in Sections 3 and 4. Unless otherwise noted, in what follows we set the parameters as $r = 0.07, \mu = 0.04, \sigma = 0.2, I = 2, Q = 0.1, \tau = 0.4, \alpha = 0.2, x = X(0) = 0.5$ (these parameters are typical). Then, we obtain Table 3. In Table 3, the investment triggers $x^i_{ae} = 3.291$ and $x^i = 2.4$ are 3.29 times as large as the zero-NPV triggers $x_{NPV} = 1$ and $\psi x_{NPV} = 0.729$, respectively. This means that the monopolist without rivals can wait for sufficiently large market demand $X(t)$. The levered firm chooses its optimal leverage $LV = 28.9$ % and invests earlier than the unlevered firm. The levered firm's investment threshold is $\psi = 0.729$ times as large as that of the unlevered firm. The unlevered and levered firms' values at the initial time are, respectively, $V_{ae}(x = 0.5) = 0.306$ and $V_{de}(x = 0.5) = 0.481$. That is, because of its optimal capital structure, the levered firm obtains 1.57 times as great a profit as the unlevered firm.

To begin with, we consider the case of a duopoly. Table 4 shows: investment triggers x^i, x_P, $\hat{x_P}$, $\tilde{x_P}$; coupons $c(x^i)$, $c(x_P)$, $c(\hat{x_P})$, $c(\tilde{x_P})$; default triggers $x^d(c(x^i))$, $x^d(c(x_P))$, $x^d(c(\hat{x_P}))$, $x^d(c(\tilde{x_P}))$; and firm values (19), (23), (25), (26). Recall that x_P, $\hat{x_P}$, and $\tilde{x_P}$ represent the preemption triggers in Duopolies (i), (ii), and (iii), respectively. Note that the unlevered firm value in Duopoly (iii) is equal to (25), i.e., 0.032.

We observe $\hat{x_P} < x_P < \tilde{x_P}$ and a large difference between $x_P = 0.763$ and $\tilde{x_P} = 1.046$ from the first row of Table 4. This property held for all other parameter values that we tried other than the example presented, although the inequality $x_P < \tilde{x_P}$ cannot be analytically proven. In fact, for most parameter settings, we observe that x_P and $\hat{x_P}$ are so close to the zero-NPV trigger ψx_{NPV} that $x_P < x_{NPV}$ by the assumption $Q_2 = 0$.

In particular, in Duopoly (iii), we see quite a large gap between the firm values of the levered and unlevered firms, though practical values of the tax rate τ always lead to Case (a) in Proposition 3. In Table 4, the levered firm's value in Duopoly (iii) is 0.301, which is 9.41 times as big as that of the unlevered firm, 0.032, and is 6.02 times as big as that of the levered firm in Duopoly (i), 0.05. Recall that the effects of the optimized capital structure in monopoly is 1.57 times. When compared with the monopoly case, whether a firm in a duopoly can afford to issue debt is a crucial point that influences the outcome.

Table 5 presents investment triggers $x^i_{(n)}$, coupons $c(x^i_{(n)})$, default triggers $x^d(c(x^i_{(n)}))$, firm values (29), and social loss (37) under competition among n levered firms for $n = 1, 2, 3, 4$. We observe from the last two rows of Table 5, that the firm value is almost completely lost and the social loss becomes nearly 100 % for $n = 4$. This is because for most parameter values we have $h^{(\gamma-\beta)} \approx 0.1 \sim 0.2$, which means $h^{4(\gamma-\beta)} \approx 10^{-4} \sim 10^{-3}$.

Finally, we show some interesting numerical comparative static results with respect to the volatility σ in the market demand $X(t)$. Table 5 compares $h^{\gamma-\beta}$ and $Loss(n)$ ($n = 2, 3, 4$) for $\sigma = 0.1, 0.2, 0.3, 0.4$ with other parameter values unchanged. In Table 5, $h^{\gamma-\beta}$ monotonically increases with σ, and by this $Loss(n)$ monotonically decreases with σ. This result is intuitive. For a higher volatility σ, firms actively enter and default. Then, the firm value of a follower who must wait for the leader's investment and default becomes higher, and the leader's incentive becomes lower. This moderates the firms' preemptive activities and makes the social loss smaller. The finding always held for all other parameter values that we tried other than the example presented.

Table 3: Values of quantities.

β	γ	h	ψ	x_{NPV}	ψx_{NPV}	x^i_{ae}	x^i	LV	CS
1.437	-2.437	1.796	0.729	1	0.729	3.291	2.4	28.9 %	1.32 %

Table 4: Comparison of a monopoly with Duopolies (i), (ii), and (iii).

	Monopoly	Duopoly (i)	Duopoly (ii)	Duopoly (iii)
Investment	2.4	0.763	0.75	1.046
Coupon	0.44	0.14	0.137	0.192
Default	1.336	0.425	0.418	0.582
Value	0.481	0.05	0.032	0.301

Table 5: n levered firms for various n.

	Monopoly	Duopoly (i)	$n = 3$	$n = 4$
Investment	2.4	0.763	0.733	0.73
Coupon	0.44	0.14	0.134	0.134
Default	1.336	0.425	0.408	0.406
Value	0.481	0.05	0.005	0.0005
$Loss(n)$	0 %	81.3 %	97.1 %	99.6 %

Table 6: $Loss(n)$ for various σ.

	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.4$
$h^{\gamma-\beta}$	0.051	0.103	0.141	0.169
$Loss(2)$	90.2 %	81.3 %	75.3 %	71.1 %
$Loss(3)$	99.3 %	97.1 %	94.9 %	92.8 %
$Loss(4)$	99.9 %	99.6 %	99 %	98.4 %

6 Conclusion

This paper investigated strategic investment with debt financing by extending the monopolist's one-growth option case in Sundaresan and Wang (2007a) to a situation allowing for the preemptive activities of several firms. We analyzed three types of duopoly, namely, (i) two symmetric firms that, whether leader or follower, can issue debt, (ii) two symmetric firms of which only the leader can issue debt, and (iii) a levered firm versus an unlevered firm. The main results for duopolies can be summarized as follows.

Unlike the competition between unlevered firms, the possibility of the leader's default generates a positive excess profit to the firms in equilibrium. In (iii) the levered firm always invests first and overwhelms the unlevered firm. The order of difficulty of the preemptive competition is then (ii), (i), (iii).

Moreover, we derived the equilibrium in oligopoly of n levered firms, and showed that the social loss from preemption increases with the number of firms. Using numerical examples, we observed that a higher volatility moderates the preemptive competition and reduces the social loss.

We now consider some interesting topics for future research. Following Sundaresan and Wang (2007a), the model in this paper does not impose any exogenous restriction on the investment cost I and the amount $K(< D(X(T), c))$ which the entrepreneur borrows by means of debt financing. In the real world, a small entrepreneurial firm that cannot issue equity is likely to have a strict restriction imposed where a part of I must be financed by debt. In a model with such a restriction we may need to reveal the effects of the competition on the leverage, although the analytical derivation of the equilibrium seems difficult. Furthermore, in a model where the debtholders are regarded as an independent player in the investment game, we may need to understand how competition among several entrepreneurs changes the allocation between entrepreneurs and debtholders.

A The equilibrium in the stopping time game

In this paper, we adopt the concept of the stopping time game introduced in Dutta and Rustichini (1993), Grenadier (1996), and Weeds (2002) for its intuitiveness. This appendix explains the equilibrium in the competition according to this concept.

First, let us consider the game between two unlevered firms in Section 3.1. The stopping time game proceeds as follows. In the absence of an action by either player, the game environment evolves according to geometric Brownian motion (1). A firm that has not invested until time t has the action set $A_t^{ae} = \{-1, 1\}$, where -1 and 1 stand for no entry and entry, respectively. If a firm has already invested before time t, then the action set A_t is the null. Entry by the leader terminates the game and determines the value of both firms since the follower necessarily takes the optimal response. In this paper, as in Weeds (2002), we assume that one of the firms can enter infinitesimally earlier than the other, even if both attempt to invest at the same timing. Also assume that the probability that a firm is chosen as a leader is fair, i.e., 1/2. A strategy for a firm is generally defined as a mapping from the history of the game H_t to the action set A_t^{ae}. The history H_t has two components: the sample pass of the process (1) and the actions of the two firms up to time t. Since the process (1) is Markovian, we restrict attention to Markovian strategies. In equilibrium, both firms then try to invest at time $\inf\{t \geq 0 \mid X(t) \geq x_{NPV}\}$, which yields $L_{ae}(x_{NPV})/2 + F_{ae}(x_{NPV})/2 = 0$ to both firms. Each firm has no incentive to deviate from the strategy because any deviation does not generate a positive payoff. Note that by the assumption, only one of the firms is allowed to enter at the preemptive timing.

Second, we consider the equilibrium of Proposition 1. The difference from the previous case only consists of the fact that a firm (shareholders) that has not invested until time t has the action set $A_t^{de} = \{-1, c \mid c \geq 0\}$, where -1 and c stands for no entry and invest along with issuing debt with coupon c, respectively. When both firms have the same strategy "invest at $\inf\{t \geq 0 \mid X(t) \geq x_P\}$ with issuing debt with coupon $c(x_P)$," any change in a firm's strategy does not generate a higher firm value. Consequently, we obtain the preemptive equilibrium of Proposition 1.

In a similar manner, we can show the preemptive equilibrium of Proposition 2. Now, consider the competition between a levered firm and an unlevered firm. Any change in one's strategy does not increase the firm value when the levered firm and unlevered firm take the strategy "invest at $\inf\{t \geq 0 \mid X(t) \geq \min\{x^i, \tilde{x_P}\}\}$ with issuing debt with coupon $c(\min\{x^i, \tilde{x_P}\})$" and the strategy "invest at $\inf\{t \geq 0 \mid X(t) > \tilde{x_P}\}$," respectively. In equilibrium, the levered firm always becomes the leader and the unlevered one invests not at $\inf\{t \geq 0 \mid X(t) > \tilde{x_P}\}$, but rather at the follower's optimal timing $T_{Fa}^{\tilde{i}}$ or $T_{Fa}^{\tilde{i}}$ of Proposition 3.

Finally, we consider the equilibrium in oligopoly, i.e., Proposition 4. As in duopoly, we assume that one of the firms can enter infinitesimally earlier than the other, even if several firms attempt to invest at the same time. We also assume that the probability that one of the firms is allowed to invest is fair.

We solve the game backwards. The subgame between the two firms after Firm 3's bankruptcy is the same as the game that we investigated in Proposition 1. By Proposition 1, in equilibrium both firms try to invest at the investment trigger $x^i_{(2)} = x_P$ with coupon $c(x^i_{(2)})$. Next consider the subgame among the three firms after Firm 4's bankruptcy. The leader who invests at $X(s)$ obtains

$$L_{de}(X(s)) = \psi^{-1}\Pi(X(s)) - I, \tag{38}$$

while the two followers obtain

$$\left(\frac{X(s)}{x^d(c(X(s)))}\right)^{\gamma} h^{\gamma-\beta} V_{de}(x^d(c(X(s)))) = h^{2(\gamma-\beta)} \left(\frac{X(s)}{x^i}\right)^{\beta} (\psi^{-1}\Pi(X(s)) - I) \tag{39}$$

if $0 < X(s) < hx^i_{(2)}$. Note that the followers obtain the firm value calculated in the subgame between the two firms after the leader's (Firm 3's) default. The solution of the equation (38)=(39) is equal to $x^i_{(3)}$ defined by (27) in Proposition 4. When all firms have the same strategy "invest at $\inf\{t \geq 0 \mid X(t) \geq x^i_{(3)}\}$ with coupon $c(x^i_{(3)})$," any change in a firm's strategy does not generate a higher firm value. By repeating the reasoning, we obtain the preemptive equilibrium described in Proposition 4.

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