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ABSTRACT

The Nikkei225 implied volatility index is constructed in this study following the model-free approach underlying the new VIX index. There are several difficulties in the implementation of this approach, ranging from the inability to identify at-the-money options to the insignificance of cumulative contributions to implied variance. These difficulties stem from the lack of market liquidity due to investors' focus on near-term options, and from asymmetric truncation that is not accommodative of sharp movements in the underlying index. It is shown that though new listing rules have the effects of decreasing the likelihood of the index falling outside the boundaries of available exercise prices, and expanding the range of reliable options in the tails of the return distribution, there remains a potential for significant truncation errors due to the CBOE filtering process. It is also shown that the correction for downward bias by lifting the filtering rules can be sensitive to maturity rollover and interpolation process.

The behavior of the Nikkei225 implied volatility index over the sample period from 1990 to 2009 is reflective of the strong impact of financial crises on market expectations about economic uncertainty. There is no strong evidence of unbiasedness and efficiency, but the implied volatility index is highly correlated with realized volatility. The empirical results suggest also that implied volatility contains some useful information beyond that conveyed by historical returns. The tests of out-of-sample forecasting accuracy also indicate that implied volatility is likely to be associated with a better average performance than asymmetric GARCH models. The model-free implied volatility index can be also useful in marginally improving the accuracy of inferences from conditional volatility. But this evidence can be sensitive to greater levels of uncertainty and higher market volatility during financial crises.

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1. Introduction

The increasing focus on market sentiment and volatility expectations during financial crises is reflective of the crucial importance of market volatility in economic policymaking and financial regulation. It is difficult however, to measure market perceptions of economic uncertainty with accuracy given the heterogeneity of volatility expectations across market participants. The reaction of investors to major events and to the release of macroeconomic information may differ over time, and their anticipated levels of uncertainty may diverge as well. Thus, an index of volatility expectations is important insofar that it aggregates anticipations of future volatility across risk-hedgers, arbitrageurs and speculators. The development of a benchmark of forward-looking volatility can be useful for market regulation and monetary policy-making purposes, among others.

The new VIX implied volatility index derived from S&P 500 options and disseminated by the Chicago Board Options Exchange is based on a model-free calculation modus that avoids measurement errors stemming from model misspecification. The index aggregates information on market volatility in the term structure of option prices. It also extracts information from the relationship of option premia with exercise prices, following an analytical approach similar to the fitting of interest rate processes to bond prices. Given the usefulness of the new VIX index as a measure of market expectations of future uncertainty, a similar Japanese volatility index based on the Nikkei 225 options is warranted as the underlying stock average constitutes an important tracking index and its derivatives instruments are traded on major Asia-Pacific derivatives markets, including the Osaka Securities Exchange, Chicago Mercantile Exchange, and Singapore Exchange.

Thus, the main purpose of this study is to introduce the model-free Nikkei225 implied volatility index for the Japanese stock market, and review the potential difficulties encountered in the implementation of the CBOE model-free methodology. It examines also the time-series properties of this new volatility index, its informational content and out-of-sample forecasting accuracy. The empirical evidence contributes to the existing literature on implied volatility, which has extended since the Black-Scholes (1973) option pricing model along several strands. It adds to the class of studies, which address the various problems encountered in the estimation of implied volatility. The numerical difficulties stem from the failure of the iterative process to equate market prices with theoretical values and from the absence of closed-form solutions to option pricing models. The model-free approach underlying the new implied volatility index can be regarded as an alternative solution to convergence problems. This methodology, as proposed by Britten-Jones and Neuberger (2000), adjusts arbitrary volatility processes to option prices, drawing upon the standard practice of fitting interest rate processes to bond prices. It is shown that the risk-neutral expectations of the integrated variance until maturity can be inferred from a set of call option prices.[1]

The present study constitutes also an attempt to contribute to the second strand of literature, which focuses on the information content of implied volatility, and its forecasting accuracy.[2] Day and Lewis (1992) provide early evidence that the volatility implicit in S&P 100 index options contains useful information for forecasting market volatility. However, it is also found to be inefficient in the sense that the conditional volatility based on GARCH modeling contains incremental information beyond that reflected by implied volatility. The empirical results from Canina and Figlewski (1993) are also suggestive of the absence of correlation between S&P 100 implied volatility and future volatility. In contrast, the empirical evidence from Jorion (1995) and Amin and Ng (1997), indicates that implied volatility provides efficient, albeit biased, estimates of future volatility in foreign exchange markets. As noted by Figlewski (1997), who provides a thorough critical discussion of the difficulties inherent to various models of volatility forecasting, the evidence of bias in implied volatility does not vitiate the rational assessment of information by market participants and any evidence on rational behavior or lack thereof, depends on tests of volatility expectations. More recent tests by Blair, Poon and Taylor (2001) using the implied volatility index from S&P 100 options, suggest that

[1] As noted by Ait-Sahalia and Lo (1998), the modeling of implied volatility as a function of exercise prices can also be based on polynomial smoothing, interpolation or splines smoothing of the pricing function.

[2] There is evidence that the information content of implied volatility can be useful in value-at-risk analysis as shown by Giot (2005), and in assessing investors' attitudes toward risk as suggested by Whaley (2000), who shows that the S&P100 implied volatility index represents a gauge of investors' fear.

implied volatility is more accurate for out-of-sample forecasting than past measures of realized volatility.[3]

In light of the existing literature, we introduce the Nikkei225 implied volatility index and describe the potential difficulties in implementing the new VIX methodology with respect to the Japanese options market. It is shown that the potential irregularities in the application of the model-free methodology stem from market liquidity problems, which are reflective of investors' interest in near-term options. They derive also from the prevailing trading rules regarding the listing of exercise prices, which imply narrow truncation intervals unable to accommodate sharp movements in the underlying index. We also assess the information content of implied volatility and its long-term relationship with realized volatility. Finally, we provide some empirical evidence on the usefulness of implied volatility for drawing inference about future uncertainty. The tests of out-of-sample forecasting accuracy are performed relative to alternative asymmetric GARCH models.

The sample period extends from 1990 through 2008 and spans several financial and economic crises, including the burst of the Japanese asset bubble, the Asian currency crisis, and the current U.S. housing and credit crisis. The results suggest that the Nikkei 225 implied volatility index is reflective of the impact of financial crises on market expectations about future uncertainty. It is also reflective of incremental information that is not contained in historical returns, and it can be associated with better out-of-sample forecasting performance than alternative forecasts based on various GARCH models that allow for asymmetric effects. However, it is also shown that its forecasting accuracy can be sensitive to greater levels of uncertainty and the onset of financial crises.

The remainder of the paper is organized as follows. The next section reviews in brief the model-free approach underlying the new VIX index calculation. Section 3 addresses the implementation issues encountered in the calculation of the Nikkei 225 implied volatility index and the approximation errors associated with this methodology.

[3] The volatility implicit in S&P 100 option prices is also found by Fleming (1998) to constitute a reliable estimate of future volatility despite its upward bias. There is evidence of efficiency in the sense that past forecast errors are orthogonal to parameters usually embedded in ARCH models. These issues are also addressed in the critical reviews of the literature on volatility forecasting is

Section 4 examines the time-series properties of implied volatility and its long-term relationship with realized volatility. Section 5 examines the information content of implied volatility with respect to various conditional volatility models. Section 6 addresses the out-of-sample performance of implied volatility and its behaviour during the U.S. credit crisis in particular. Section 7 concludes the paper.

2. Model-free implied volatility index calculation

The new VIX index is disseminated by the Chicago Board Options Exchange and differs from the original VIX in many ways. The latter is based on the S&P100 options and it uses the Black-Scholes (1973) model to derive implied volatilities from at-the-money and near-the-money options over the near-term and next-term options. In contrast, the new VIX is based on the S&P500 index and it employs a model-free approximation for a hypothetical option with exactly one month remaining to expiration, and with exercise price equal to the theoretical forward level. This model-free approach has the merit of gathering information contained not only in near-the-money options, but in the broader volatility structure, including out-of-the-money call and put options. The calculation modus is based on the following expression of implied variance

$$v^2 = \frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} e^{rT} \; Q(T, K_i) - \frac{1}{T}\left(\frac{F}{K_0} - 1\right)^2, \tag{1}$$

where T is the time remaining to expiration and r refers to the risk-free interest rate. The forward index level, with maturity matching that of the hypothetical option can be determined from the put-call parity relationship as $F = K^* + e^{rT}(C^* - P^*)$. The exercise price K^* corresponds to the listed strike price for which the difference between at-the-money call and put prices $(C^* - P^*)$ is minimal. The exercise price K_0 in Equation (1) is set to equal to the forward level F or falls immediately below it.

As noted by Jiang and Tian (2007), this model-free approach is based on the concept of fair value of future variance, which can be extracted directly from option prices following Demeterfi, Derman, Kamal and Zou (1999)

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provided by Figlewski (1997) and Poon and Granger (2003), among others.

$$v^2 = \frac{2}{T}\left\{ rT - \left[\frac{S_0}{S_*}e^{rT} - 1\right] - \ln\left(S_*/S_0\right) + e^{rT}\int_0^{S_*}\frac{P(T,K)}{K^2}dK + e^{rT}\int_{S_*}^{\infty}\frac{C(T,K)}{K^2}dK \right\}, \qquad (2)$$

where S_0 is the current stock price while S_* represents an arbitrary stock price close to the forward level. The concept of fair value of future variance is also shown to be identical to the model-free implied variance introduced by Britten-Jones and Neuberger (2000).[4] Jiang and Tian (2007) provide evidence of the theoretical consistency of the new VIX index with the model-free implied variance.

The new VIX methodology uses only out-of-the-money put $(K_i < F)$ and call $(K_i > F)$ options. Upon ranking the listed strike prices in ascending order, only reliable options with non-zero bid prices are selected. As put (call) options are spanned from K_0 to the lower exercise price K_L (upper exercise price K_U), the application of the CBOE truncation process implies that option selection is discontinued upon encountering two consecutive options with zero-bid prices. The spread between exercise prices is determined as the average difference between the closest strike prices $\Delta K_i = \left(K_{i+1} - K_{i-1}\right)/2$. At the lower and upper limits of the spectrum of exercise prices however, ΔK is estimated as the difference between the exercise price at the limit and its adjacent strike. The midpoint of bid-ask quotes $Q(.)$ is used to calculate the marginal contribution of each option to the model-free implied variance $\frac{\Delta K_i}{K_i^2}e^{rT}Q\left(T,K_i\right)$, which is function of exercise prices and option premia. With respect to the exercise price K_0, the average of call and put prices is used to estimate the hypothetical option premium and the contribution to implied variance.

The cumulative contributions $\sum_i \left(\Delta K_i/K_i^2\right)e^{rT}Q(T,K_i)$ can be determined as the sum of individual options contributions, which depends on the drawing of truncation lines. The implied variance described by equation (1) can thus be calculated by subtracting the term $\frac{1}{T}\left(\frac{F}{K_0} - 1\right)^2$ in order to adjust for the bias in using in-the-money call with respect

[4] The concept of model-free implied variance derives from the development of variance swaps by Dupire (1994), and Neuberger (1994) and revisited by Carr and Wu (2004) and Jiang and Tian (2005), inter alia.

to K_0. Using the estimates of implied variance for the nearest maturity v_1^2 and the next expiration v_2^2, it is possible to implement the maturity interpolation process described in Equation (3) to derive the implied variance for the theoretical option with one-month remaining to expiration

$$v^2 = \left(T_1 v_1^2 \left[\frac{N_{T_2} - N_v}{N_{T_2} - N_{T_1}} \right] + T_2 v_2^2 \left[\frac{N_v - N_{T_1}}{N_{T_2} - N_{T_1}} \right] \right) \times \frac{N_y}{N_v}, \tag{3}$$

where T_1 and T_2 denote the time remaining to the near and next-term maturities. Also, N_T represents the number of minutes until expiration, while N_v and N_y refer to the number of minutes during the periods of one month and one year, respectively. Finally, the model-free implied volatility index is calculated as the square root of the implied variance v^2, and expressed in percent terms.

3. Implementation issues and approximation errors

The model-free Nikkei225 implied volatility index is constructed following the CBOE procedure, using the closing prices of the corresponding options traded on the Osaka Securities Exchange. The inception of trading on the European-type Nikkei225 index options started in June 12, 1989, and the daily option prices are obtained from the Nikkei Needs FinancialQuest database. The average bid-ask yields on the three-month certificates of deposit used as proxy for the short-term interest rates are obtained from Thomson Reuters database The sample of daily observations covers a period of twenty years starting from January 1990 to December 2009, and spanning 240 monthly options expirations.

There are some difficulties in the implementation of the new VIX procedure, which range from the inability to identify at-the-money options to the insignificance of cumulative contributions to implied variance. These difficulties stem essentially from the limited spectrum of listed exercise prices, truncation rules and market liquidity problems. The underlying assumption that the index evolves within the upper and lower boundaries of listed strike prices is not necessarily satisfied. Indeed, for the near-term maturities over the sample period, the range of exercise prices lower than the index $S_0 - K_L$ is on average

1.40 times the range of strike prices above the index $K_U - S_0$, while the average range amounts to 1.37 for the next-term maturities. This bias toward lower strike prices can be indicative of the relative emphasis of market participants on risk-hedging positions. This measure of asymmetric truncation can however, take as low a ratio as 0.0035 (August 3, 1990) when the index drifted below the minimum exercise price available K_L. It can also be as high as 83.60 (August 15, 1995) when the index rises sharply above the highest listed strike K_U.[5]

These asymmetric truncations can thus be due to sudden and significant changes in the underlying index and the trading rules regarding the listing of strike prices. The exercise prices of newly traded contracts are listed in multiples of 500 yen and until February 2006, additional strike prices were listed with similar intervals. Changes in the listing rules introduced to render the options market more accommodative of sharp index fluctuations, allow also for a better partition of exercise prices for risk-hedging purposes. Indeed, additional strike prices are recently listed for the three nearest contract months with 250-yen intervals. The new listing rules applicable from September 2008 effectively reduced the minimum difference between strike prices ΔK_i to 250 yen, irrespective of the prevailing level of Nikkei 225 spot prices. [6] This has the potential of reducing discretization errors, which are due as noted by Jiang and Tian (2007), to the numerical approximation of the integrals $\int_{K_L}^{K_0} \frac{P(T,K)}{K^2} dK + \int_{K_0}^{K_U} \frac{C(T,K)}{K^2} dK$ with the actual partition $\frac{\Delta K_i}{K_i^2} e^{rT} Q(T, K_i)$ based on the available range of listed strike prices. The better partition of strike prices has also the potential of reducing the gap between the forward level and the exercise price immediately below it. It can also contribute toward the reduction of the downward bias from negative expansion errors, which are due to the Taylor series

[5] The index closed below the lower boundaries of listed exercise prices in 17 cases, 13 of which occurred in 1990 alone, as the listing of new exercise prices did not adjust adequately to the burst of the Japanese asset bubble. There is a failure to accommodate for sharp decreases in the index in fewer cases, once in 1991 and 1995 and twice in 1997. In contrast, the index closed beyond the upper boundaries of listed strike prices in four cases between 1991 and 1995.

[6] An earlier amendment to trading rules with effect from March 2006, requires additional strike prices for the three nearest month contracts to be listed with 250-yen intervals under the condition that the spot price falls below 10,000 yen.

approximation of the log function $\ln(F/K_0)$.

The addition of strike prices widens the truncation interval over time depending on the daily changes in the index, diminishing thereby the likelihood of the index suddenly falling outside the range of available exercise prices. However, it does not necessarily eliminate the negative truncation errors $-\frac{2}{T}e^{rT}\left\{\int_0^{K_L}\frac{P(T,K)}{K^2}dK+\int_{K_U}^{\infty}\frac{C(T,K)}{K^2}dK\right\}$, which are due to the approximation of the infinite range of exercise prices with the finite range (K_L, K_U). It is noted in particular, that the new VIX procedure discards lower (higher) exercise prices after encountering two consecutive puts (calls) with zero-bid prices. The objective of this filtering process is to avoid illiquid and mispriced options, but it can also introduce a downward bias in the estimation of implied volatility. The suspension of filtering rules is not always conducive to the correction of downward bias as the aggregate approximation errors may be negative. The inclusion of options, otherwise discarded, has indeed the effect of increasing the cumulative contributions $\sum_i(\Delta K_i/K_i^2)e^{rT}Q(T,K_i)$, but despite this increase, it can be shown that the maturity rollover and interpolation process can ultimately lead to lower estimates of implied variance.[7]

It is noted that as expiration draws near, the rollover to the second and third contract months takes place with eight days remaining to maturity in order to avoid pricing errors from options with imminent expiration.[8] This rollover process can result in both the near-term and next-term maturities falling farther beyond the hypothetical 30-day horizon underlying the implied volatility index. The extrapolation process has in this case, the effect of driving the implied volatility index lower because, or despite, the increase in next-term implied variance as the term $N_v - N_{T_1}$ in Equation (3) takes a negative value. In

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[7] In the cases where the estimation of implied volatility is sensitive to filtering rules, the widening of the truncation interval can indeed lead to higher estimates of implied volatility (correction of downward bias) in 77% of the cases, with the correction of truncation errors amounting to an average of 0.33%. It is however, conducive to lower volatility estimates in the remaining cases, with 0.92% difference on average.

[8] The expiration date for the OSE-traded Nikkei225 index options is the second Friday of each maturity month. The last trading date is the day preceding option expiry. It is also noted that the options market closes at 3:10 pm, whereas trading in the underlying index component stocks

fact, this interpolation process assumes that the term structure of implied variance is linear in option maturity. As this is not necessarily the case, the new VIX methodology can be conducive also to interpolation errors, which add to the discretization, expansion and truncation errors discussed above.

Thus, given the CBOE procedure and options market structure, some irregularities appear when the minimum spread between call and put prices cannot be calculated due to the absence of at-the-money options with non-zero prices. This in turn implies that the forward level cannot be determined either. Also, it may not be possible to identify the exercise price K_0, immediately below or equal to the forward index level, due to the limited range of listed options. Even when these initial steps in the estimation procedure do not pose difficulties, there is still a possibility that the cumulative contributions to implied variance amount to nil because of illiquid options and the application of truncation rules.

There are few irregularities encountered in the estimation of implied variance for either or both the near-term and next-term maturities over the full sample period from 1990 to 2009. It is noted that 98% of such cases occurred in the early trades until 1997. It is the estimation of implied variance for next-term maturities that is almost exclusively associated with irregularities stemming from the inability to identify at-the-money options. This is possibly attributable to problems of market liquidity in the early 1990s, and it can also be reflective of a tendency for investors to focus on options with short-horizon maturities. It is also noted that the near-term options are rather associated with difficulties in identifying the exercise price below or equal to the forward index level. These difficulties can be due instead to the narrow spectrum of listed strike prices. Finally, the filtering rules and lack of liquid options with longer expirations can be conducive to zero-cumulative contributions, difficulties that are again encountered essentially with respect to next-term options.

When such difficulties are encountered with respect to the near-term or next-term maturities, implied variance is approximated using estimates from the previous date. It is only when implied variance cannot be estimated for both the near-term and next-term

closes ten minutes earlier.

maturities that the implied volatility index is set equal to its estimate from the previous day. These limited adjustments are warranted in order to construct the time-series of Nikkei225 implied volatility index, the properties of which can be thus compared to the new VIX index.

4. Time series properties and relationship with realized volatility

The time-series behavior of the Nikkei225 implied volatility index is described by Figure 1, relative to the new VIX index, which is obtained from Thomson Reuters database. It is clear that the degree of uncertainty anticipated by market participants varies over time. The level of volatility expectations in the Japanese market remain higher than in the U.S. market in the first half of the 1990s. This tendency is reflective of the effects of the Japanese post-bubble recession and the heightened level of uncertainty about macroeconomic policies, deflationary pressures and financial stability. There are also anticipations of higher volatility in association with other major events such as the Russian debt default and LTCM crisis in 1998. The inception of the Euro currency unit in 1999 does not however, seem to be associated with perceptions of greater uncertainty, and the burst of the information technology bubble in 2000 seems to have a limited impact on the level of anticipated uncertainty. However, the onset of the U.S. housing and credit crisis toward the end of 2007 has the effect of driving volatility expectations significantly higher. There is indeed a sharp increase in both implied volatility indices, reaching unprecedented levels toward the end of 2008. The behavior of volatility expectations in these markets seems to follow similar patterns of jumps in association with financial crises, which are followed by monotonous declines afterward.

As noted earlier, the estimation of implied volatility using the Nikkei225 options prices can be sensitive to the implementation of truncation rules. The suspension of the truncation process is conducive to non-zero truncation errors in 9.3% of sample observations from 1990 to 2009. The mean absolute value of truncation errors amounts to 0.465%, with an average downward bias of -0.049%. It is clear from Figure 2 that such truncation errors are particularly frequent in early options trades, with two-thirds of these cases occurring by the end of 1996 and with fewer significant errors as recent as December

2008.

The distributional moments reported in Table 1 indicate that the Nikkei225 implied volatility index is on average higher than the new VIX index. The higher volatility expectations for the Japanese market are reflective of the prolonged post-bubble period of economic recession, among other factors. Judging from the sample standard deviations, it is also noted that the levels of fluctuations in implied volatility are rather close. The evidence from the distribution of the first differences in implied volatility indicates that the dynamics of volatility expectations can change significantly under regimes of higher uncertainty. The US financial crisis, which is broadly defined over the three-year period from 2007 to 2009, has the effect of increasing the perceived degree of uncertainty and driving implied volatility to unprecedented levels, significantly higher than the long-term mean. It can also render volatility expectations themselves, more volatile. Judging from the results of unit-root tests, the time-series of implied volatility during the financial crisis do not tend to be stationary.

In order to assess the long-term relationship between the implied volatility index and observed levels of market volatility, ex-post non-parametric estimates of realized volatility are used. In order to reduce errors in the estimation of the true mean, realized volatility is measured following Figlewski (1997), as the sample standard deviation of daily squared returns z^2, under the assumption of zero-mean distribution.

$$RV_t = \sqrt{\tfrac{1}{(n-1)} \sum_{s=t}^{t+n} z_s^2} \tag{4}$$

Though longer historical samples can yield more accurate estimates, this equation restricts return realizations to the exact fixed period of $n = 30$ days, which is consistent with the time to expiration of the hypothetical option underlying the new VIX methodology. This *ex post* estimate of realized volatility is expressed on an annualized basis, and in percentage terms for the sake of comparison with the implied volatility index. It is clear from Table 2 that the relationship between implied and realized volatilities is indeed positive. However, it seems that implied volatility does not constitute a perfect forecast of market volatility, as the regression line differs from that characteristic of unbiased forecasts. There is also evidence of higher volatility levels for the Japanese market, judging from the greater dispersion in the implied and realized volatility series.

As in Christensen and Prabhala (1998), the informational efficiency of implied volatility is typically assessed by regressing realized volatility on implied volatility as follows

$$RV_t = \delta_0 + \delta_{iv} IV_t + \zeta_t, \quad (5)$$

where RV_t denotes the realized volatility, defined above using return realizations over the period $[t, t+30]$, while IV_t denotes implied volatility at the beginning of the one-month period. The implied volatility index can be regarded as an efficient estimate of realized volatility if the residuals ζ_t are white noise. It constitutes rather an unbiased forecast of realized volatility in the presence of evidence that δ_0=0 and δ_{iv} =1.

The regression equation (5) is estimated using Newey-West heteroskedasticity consistent coefficient covariances to obtain unbiased estimates of standard errors. The results reported in Panel A of Table 3 using daily observations indicate that the null that these coefficients are respectively equal to zero and unity, is rejected judging from the F-statistics for Wald tests. Also, Lagrange Multiplier (LM) tests indicate the presence of serial correlation in the residuals, which suggests that implied volatility does not constitute an efficient estimate of market volatility. It is noted that the addition of autoregressive terms affects the sign and significance of the intercept and slope coefficients, and only Ljung-Box statistics do not suggest the presence of serial correlation in the residuals. Wald tests of the null hypothesis including restrictions on the significance of autoregressive terms also reject the unbiasedness of implied volatility. Arguably, the evidence of positive relationship can be, as noted by Amin and Ng (1997), the artifact of the measurement approach resulting in overlapping observations of realized volatility. In order to reduce measurement errors and avoid inconsistent estimators, the regression model is reexamined using monthly non-overlapping observations. The estimates of realized volatility until option expiration and implied volatility are gathered with respect to monthly maturities spanning the period from February 1990 to December 2009, resulting in a sample of 239 non-overlapping observations.

The model estimates based on monthly expirations reported in Panel B of Table 3, stand in sharp contrast with those obtained from daily observations. Indeed, the intercepts tend to be insignificant while δ_{iv} coefficients for implied volatility are found to be positive and significant. With the addition of lagged values of realized volatility, the coefficients for implied volatility remain significant and close to unity, and judging from LM tests, there is no indication of serial correlation in the residuals. The Wald tests for these model estimates fail to reject the joint hypothesis restricting the intercept and δ_i coefficient to zero and unity respectively, lending support to the proposition of unbiasedness in implied volatility. Thus, the evidence from monthly expirations that avoid measurement problems associated with overlapping volatility observations suggests that implied volatility is not an efficient estimate of market volatility, but it lends some support for its unbiasedness.

5. The incremental information content of implied volatility

The important question remains as to whether implied volatility contains useful information that is not conveyed by conditional volatility from GARCH models, which have the merit of capturing different patterns of volatility clustering and allow for asymmetric effects. Under the parsimonious GARCH model, it is possible to express market returns $z_t = \mu + \xi_t$ as a function of drift μ and error terms ξ_t, conditional on the information set $\Im_{t-1}$ available at time $t-1$, and the conditional variance can be expressed as follows

$$\sigma_t^2 = w + \beta\, \sigma_{t-1}^2 + \alpha\, \xi_{t-1}^2 \tag{6}$$

It can be shown that when $\beta > 0$ and $\beta + \alpha < 1$, the conditions for positive definiteness and stationarity of the variance are satisfied. In order to assess the incremental information content of implied volatility, it is possible following Day and Lewis (1992) and Lamoureux and Lastrapes (1993), to include the lagged estimates of implied variance into the conditional variance equation

$$\sigma_t^2 = w + \alpha\, \xi_{t-1}^2 + \beta\, \sigma_{t-1}^2 + \pi\, IV_{t-1}^2 \tag{7}$$

In this augmented version, there is evidence that the implied volatility index contains useful information beyond that reflected by past variance forecast and historical returns if the null of $\pi = 0$ is rejected while $\alpha = \beta = 0$. The implied volatility index would subsume the information contained in historical returns if π approaches unity while α and β coefficients are found to be insignificant.

It is also important, in light of the growing literature on leverage effects, to account for the asymmetric impact of news on market volatility. The augmented version of the exponential GARCH model by Nelson (1991) assumes a generalized error distribution structure for the residuals and expresses the conditional variance as follows

$$\log(\sigma_t^2) = w + \alpha \left| \frac{\xi_{t-1}}{\sigma_{t-1}} \right| + \beta \log(\sigma_{t-1}^2) + \gamma \frac{\xi_{t-1}}{\sigma_{t-1}} + \pi \log(IV_{t-1}^2) \tag{8}$$

Another competing model introduced by Glosten, Jagannathan and Runkle (1993), can also be used to account for asymmetries in the volatility dynamics without assuming exponential leverage effects. The conditional variance captures asymmetries in the reaction to shocks to the return-generating process, and it can be expressed as a function of observed implied volatility as follows

$$\sigma_t^2 = w + \alpha\, \xi_{t-1}^2 + \beta\, \sigma_{t-1}^2 + \lambda\, r_{t-1}^2 I_{t-1} + \pi\, IV_{t-1}^2 \tag{9}$$

The asymmetry indicator I_{t-1} is set equal to unity in the case of negative returns $r_{t-1} < 0$ and zero otherwise. The incremental information content of implied volatility can be examined on the basis of the magnitude and sign of π coefficients in these models.

The estimation period extends from January 1990 to December 2009, and includes the U.S. credit crisis where high levels of implied volatility were observed. The various GARCH models are estimated in their standard versions as well as those including past levels of implied volatility as regressors For the sake of brevity, only the results for those augmented with lagged implied volatility (Model-IV) are reported in Table 3. It is clear that these models can capture the dynamics of market volatility when past observations of implied volatility are used as volatility regressors. This evidence is robust to variations in model specifications and discrepancies across implied volatility indices. For the Japanese market, the estimated β coefficients are found to be positive and significant, while the sign and magnitude of α coefficients for ARCH terms depend on model estimation. Higher-order GARCH models are estimated in order to account for any remaining ARCH effects. The negative sign associated with first-order ARCH terms can be indicative of the suppressing effects on more distant shocks, which rather tend to elevate market volatility.

The estimation results provide also evidence of significant leverage effects, which are reflected by the negative γ coefficients in exponential GARCH models and positive λ coefficients in GJR-GARCH models. It is noted that the lagged value of implied volatility included into the variance equation, is always associated with a significant π coefficient, irrespective of volatility models and markets. The positive sign suggests that conditional volatility is an increasing function of the observed levels of implied volatility. This evidence suggests that implied volatility contains some incremental information that is not reflected by historical returns. However, the significance of the remaining parameters including ARCH and leverage effects also indicates that implied volatility does not convey all information about market volatility, and as such, it does not subsume the information contained in historical returns.

6. Model-free inference

In light of the evidence on the informational content of implied volatility, it is also important to assess its economic significance for out-of-sample forecasting. The forecasting accuracy is measured relative to inferences from conditional variance. Because GARCH models are not designed for multi-step forecasting purposes however, the inferences are based on one-step-ahead static forecasts instead. The estimation is based on the various volatility models described above, including both the unrestricted versions where past estimates of implied volatility are introduced as variance regressors. These models are estimated using observations within rolling periods of ten years. Using about 2170 daily observations in each period, it is possible to achieve robust estimation from these ARCH-type models, which require large sample observations.

With respect to each estimation period, one-step-ahead static forecasts are generated for out-of-sample subsequent periods of six months, including approximately 130 days each. Thus, for the initial ten-year estimation period extends from January 1990 until December 1999, out-of-sample static forecasts cover the first half of 2000. In order to account for possible changes in the underlying volatility structure, this process is repeated using estimation periods starting six months later. There are no adjustments made to account for unusual events though. As a result, the entire out-of-sample period extends from January 2000 to December 2009. For the purposes of comparison with the implied volatility index, conditional volatility is computed as the annualized square root of conditional variance and expressed in percentage terms.

Forecast errors are computed as the spread between the implied volatility index and realized volatility $\psi_t^i \equiv IV_t - RV_t$, where realized volatility is measured as the annualized standard deviation using return realizations over the period $[t, t+30]$. Similar measures of forecast errors $\psi_t^c \equiv \sigma_t - RV_t$ can be obtained with respect to the annualized conditional volatility from various GARCH models. The forecasting accuracy is assessed using the root mean squared errors (RMSE), mean absolute errors (MAE) and the frequency of volatility overestimation, representing the upward bias.[9] It appears from Panel A of Table 4, which reports the performance measures for the forecast period from 2000 to 2006, that the model-free implied volatility index is more likely to be associated with lower MAE estimates. The evidence from RMSE measures suggests also that the implied volatility is on average, associated with a better out-of-sample forecasting accuracy than inferences based on conditional volatility. It is also possible to examine whether the inclusion of lagged estimates of implied volatility as variance regressors is conducive to marginal improvements in the forecasting performance of GARCH models. The evidence suggests that inferences based on these augmented-models are likely to be associated with lower RMSE and MAE estimates.

Another measure of forecast accuracy is provided by the sign of forecast bias, or the frequency of positive forecast errors ψ. There is evidence of upward bias for the Nikkei 225 implied volatility index and the new VIX index. It is however comparable to the upward bias in inferences based on GARCH models, which assume variance stationarity and are more sensitive to large deviations of returns from the long-run mean. The inclusion of implied variances in GARCH models does not whittle down the significance of the upward bias, as this process can have the counterproductive effect of

[9] The calculation of standard deviation for RMSE estimates for the purposes of assessing the statistical significance may not be appropriate because, as noted by Figlewski (1997), standard errors are likely to be biased due to the autocorrelation of forecast errors.

introducing a downward bias instead.

The issue remains as to whether this relative forecasting performance is robust to elevated levels of uncertainty such as those experienced during the U.S. financial crisis, which is broadly defined over the three years period from 2007 to 2009.[10] The measures of forecasting accuracy reported in Panel B of Table 4 indicate a significant deterioration in forecasting performance. The unprecedented levels of uncertainty during the crisis have not been fully anticipated. Instead of upward bias, there is rather evidence of a significant underestimation of market volatility based on the Nikkei225 implied volatility index and the new VIX index. The downward bias is also characteristic of inferences based on GARCH modeling, particularly with respect to the U.S. market. Thus, there is clear evidence that forecast errors are more significant during periods of higher market volatility. Perceptions of greater uncertainty may induce significant changes in the volatility structure and affect the accuracy of inferences based on implied volatility and GARCH models.

The model-free implied volatility index is generally found to be associated, despite its upward bias, with better predictive power and the potential to marginally improve the performance of conditional volatility. Part of the explanation for this significant forecasting ability may lie inter alia, in the absence of GARCH-like restrictions on variance stationarity, and the model-free approach that aggregates expectations about market volatility without suppressing any particular features of the volatility dynamics. It can be also attributed to the underlying forward-looking approach to the formation of volatility expectations that can capture information beyond that contained in historical returns.

[10] With respect to this broader definition of the onset of the U.S. financial crisis, it is noted that the Federal Reserve Bank of Saint Louis provides a timeline of the crisis starting with the announcement of Freddie Mac that it would no longer buy the most risky subprime mortgages and mortgage-related securities. It is however over the three-month period from September to November 2008, that equity markets reflected perceptions of higher uncertainty with unprecedented levels of volatility. Both the Nikkei225 and SP500 indices dropped by more than 30% while average annualized realized volatility rose above the 85% level over this period. The downward bias of the Nikkei225 and new VIX implied volatility indices during the three months of the crisis, is clearly reflected by average forecast errors of -29% and -35%, respectively.

7. Conclusion

The new Nikkei225 implied volatility index introduced in this study is constructed according to the model-free methodology underlying the new VIX index. There are various approximation errors induced by this approach as well as irregularities in its implementation using the closing prices of the Nikkei225 options traded on the Osaka Securities Exchange. It is shown that difficulties, ranging from the inability to identify at-the-money options to the insignificance of cumulative contributions to implied variance, are encountered essentially with respect to options trades in the early 1990s. The implementation difficulties are mainly due to the listing rules for exercise prices, market liquidity problems, and the CBOE filtering rules, which are conducive to truncation errors. Part of the problem stems from investors' focus on near-term options, the limited spectrum of listed exercise prices, and asymmetric truncation, which is not always accommodative of sharp movements in the index.

The behavior of the Nikkei225 implied volatility index provides evidence on market anticipations of greater uncertainty in association with major economic events and financial crises. It is shown that volatility expectations are reflective of the changing level of uncertainty over time. The empirical results provide also some insights into the information content of implied volatility and its long-term relationship with realized volatility. Despite its upward bias, the implied volatility index is found to convey useful information on future market volatility beyond that contained in historical returns. The tests of efficiency suggest that implied volatility reflects some but not all information about the future level of uncertainty and price fluctuations. Also, out-of-sample forecasting tests indicate that implied volatility is likely to be associated with a better average performance than inferences from GARCH models. The incremental information of model-free implied volatility can be also useful in marginally improving the forecasting accuracy of conditional volatility. However, the forecasting accuracy of implied volatility can be also sensitive to higher levels of uncertainty, particularly during financial crises, where GARCH models can become more reliable due to their higher sensitivity to large deviations from mean returns.

Given the above evidence on the forward-looking properties of the Nikkei225 implied volatility index relative to the new VIX index, it is clear that this model-free benchmark of volatility expectations constitutes a useful aggregate of market beliefs and perceptions about future uncertainty. It has the potential of providing useful insights into important issues such as the contagion effects of financial crises and market reaction to macroeconomic information. It can also provide some evidence that is germane to the debate over excessive volatility and financial market regulation.

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Figure 1- The behavior of model-free implied volatility index (sample period 1990~2009)

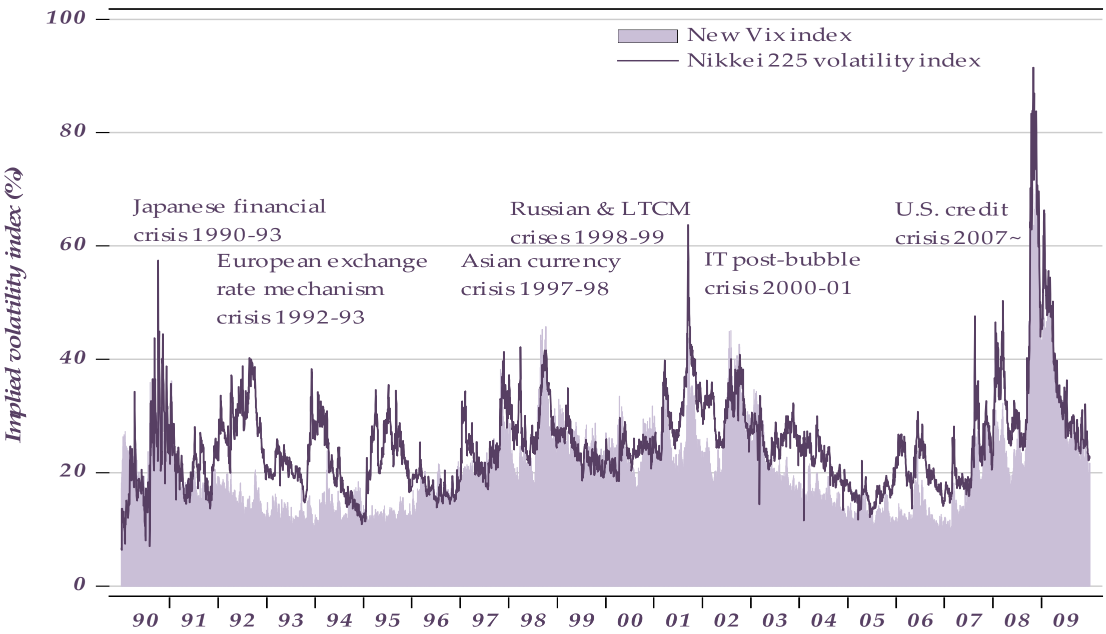

Figure 2. Truncation errors in the estimation of the Nikkei225 implied volatility index (sample period 1990 ~2009)

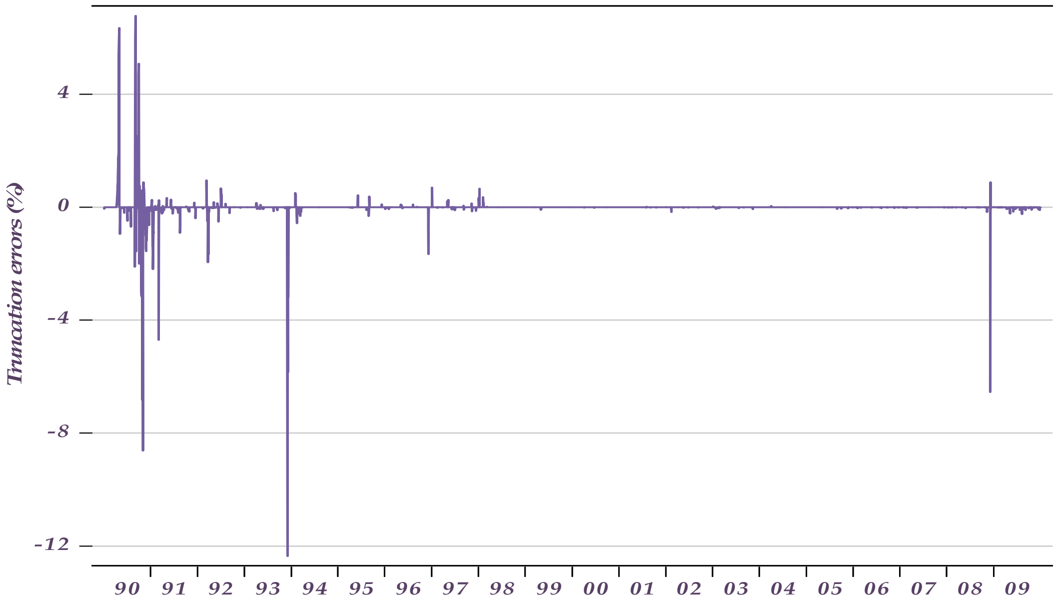

Table 1. Distributional moments and stationarity tests

Distributional moments	Japanese Markets			U.S. Markets		
	Nikkei 225 volatility		Nikkei 225	New VIX		S&P 500
	Levels	First differences	Returns	Levels	First differences	Returns
Mean						
1990-2009	25.17	0.003	-0.025	20.27	0.004	0.022
2007-2009	32.52	0.010	-0.063	27.19	0.013	-0.031
Minimum						
1990-2009	6.36	-21.280	-12.111	9.31	-17.360	-9.470
2007-2009	14.63	-10.780	-12.111	9.89	-17.360	-9.470
Maximum						
1990-2009	91.45	34.950	13.235	80.86	17.240	10.957
2007-2009	91.45	17.670	13.235	80.86	16.540	10.957
Std. Dev.						
1990-2009	8.76	1.893	1.535	8.33	1.475	1.153
2007-2009	14.62	2.525	2.016	13.13	2.501	1.853
Skewness						
1990-2009	2.294	2.378	-0.021	2.030	0.706	-0.199
2007-2009	1.634	1.071	-0.374	1.461	0.212	-0.175
Kurtosis						
1990-2009	13.115	45.904	8.736	10.171	25.203	12.582
2007-2009	5.639	10.879	10.200	5.103	14.751	9.410
ADF tests						
1990-2009	-5.548 ^{b*}	-30.608 ^{c*}	-54.583 ^{c*}	-4.825 ^{b*}	-24.798 ^{c*}	-30.097 ^{c*}
2007-2009	-1.893 ^{b}	-19.983 ^{c*}	-21.545 ^{c*}	-1.938 ^{b}	-17.253 ^{c*}	-23.781 ^{c*}

Notes: The full sample period extends from January 1990 to December 2009, resulting in 5218 daily observations. The U.S. credit crisis is defined over the three-year period from 2007 to 2009 including 784 observations). ADF tests are the Augmented Dickey-Fuller test statistics, using Schwarz information criterion with additional lags included to eliminate any remaining ARCH effects. The stationarity tests with both intercept and trend, with intercept only, and with neither intercept nor trend terms, are denoted by $^{a,\ b}$ and ^{c}, respectively. *, **, *** indicate significance at the 1, 5 and 10% levels respectively.

Table 2. Modelling the long-term relation between implied and realized volatilities

Regression parameters	δ_0	δ_1	δ_{iv}	LB	LM	Adjusted R^2	Wald Test
Panel A				Daily observations			
Nikkei 225	0.529 [a]	1.006 [a]	-0.013 [c]	0.277	6.344	0.983	14285.754
	(0.000)	(0.000)	(0.053)		(0.000)		(0.000)
Period A	0.658 [a]	1.022 [a]	-0.016 [b]	0.264	4.361	0.975	14761.801
	(0.000)	(0.000)	(0.033)		(0.000)		(0.000)
Period B	0.601 [a]	0.945 [a]	-0.017	0.391	3.527	0.992	4885.696
	(0.001)	(0.000)	(0.165)		(0.000)		(0.000)
S&P 500	0.193 [a]	1.019 [a]	-0.001	0.642	4.272	0.989	14423.581
	(0.002)	(0.000)	(0.845)		(0.000)		(0.000)
Period A	0.180 [a]	1.011 [a]	0.005	0.273	24.297	0.981	15266.839
	(0.001)	(0.000)	(0.450)		(0.000)		(0.000)
Period B	0.387 [b]	1.014 [a]	-0.010	0.165	5.753	0.992	4047.285
	(0.022)	(0.000)	(0.513)		(0.000)		(0.000)
Panel B				Non-overlapping monthly observations			
Nikkei 225	4.409 [b]	0.144	1.002 [a]	0.382	2.391	0.538	4.036
	(0.014)	(0.139)	(0.000)		(0.010)		(0.019)
Period A	4.247 [c]	0.180 [c]	0.900 [a]	0.814	1.664	0.404	1.854
	(0.077)	(0.066)	(0.000)		(0.092)		(0.159)
Period B	3.014	0.025	1.249 [a]	0.457	0.532	0.684	1.108
	(0.327)	(0.893)	(0.000)		(0.849)		(0.343)
S&P 500	-1.197	0.283	0.835 [a]	0.883	2.349	0.658	4.826
	(0.180)	(0.117)	(0.000)		(0.012)		(0.009)
Period A	-1.325	0.109	0.888 [a]	0.647	0.973	0.618	2.631
	(0.396)	(0.313)	(0.000)		(0.469)		(0.075)
Period B	3.868	0.469	0.894 [c]	0.975	0.375	0.639	0.900
	(0.271)	(0.239)	(0.058)		(0.945)		(0.417)

Notes: The regression model is estimated using the Newey-West heteroskedasticity consistent coefficient covariances. Realized volatility is defined as the annualized standard deviation of squared returns over the rolling period of 30 days. The sample period of daily implied volatility estimates extends from January 1990 to December 2009, while that of monthly observations includes 239 option expirations from February 1990 to December 2009. Period A runs from 1990 to 2006 while Period B covers the U.S. crisis, which is broadly defined over the following three years from 2007 to 2009. Additional autoregressive terms are included to account for serial correlation in the residuals, but only the coefficients of first autoregressive terms are reported here for the sake of brevity. Significance at the 1, 5 and 10% level is denoted by the superscripts [a], [b] and [c], respectively and asymptotic p-values are reported in brackets. LB denotes the asymptotic *p*-values for Ljung-Box Q-statistics distributed as χ^2 on the null of no serial correlation up to the 10th lag. LM refers to the Lagrange Multiplier test for serial correlation in the residuals up to the 10th order. F-statistics are reported for Wald test of the null $(\delta_0 = 0, \delta_{iv} = 1)$, and the associated *p*-values are shown in brackets.

Table 3. The incremental information in implied volatility indexes

Model Parameters	GARCH models		Exponential GARCH models		GJR-GARCH models	
	Nikkei225	S&P500	Nikkei225	S&P500	Nikkei225	S&P500
μ	0.002	0.027	-0.029	0.016	-0.034	0.015
	(0.928)	(0.018)	(0.080)	(0.129)	(0.044)	(0.179)
w	0.025	-0.049	-0.954	-1.505	0.009	0.002
	(0.441)	(0.145)	(0.000)	(0.000)	(0.665)	(0.759)
α_1	0.097	-0.007	-0.027	-0.129	-0.029	-0.063
	(0.000)	(0.812)	(0.554)	(0.002)	(0.025)	(0.000)
α_2		0.016	0.166	0.117	0.020	0.011
		(0.362)	(0.000)	(0.024)	(0.194)	(0.500)
α_3		0.008		0.085		0.030
		(0.642)		(0.017))		(0.031)
α_4		0.033				
		(0.061				
β	0.720	0.217	0.838	0.784	0.720	0.845
	(0.000)	(0.560)	(0.000)	(0.000)	(0.000)	(0.000)
γ			-0.169	-0.179		
			(0.000)	(0.000)		
λ					0.207	0.166
					(0.000)	(0.000)
π	0.001	0.002	0.148	0.238	0.001	0.000
	(0.000)	(0.043)	(0.000)	(0.000)	(0.000)	(0.000)
LB	0.727	0.181	0.584	0.258	0.720	0.278
LM	0.142	0.110	0.181	0.135	0.163	0.240
LL	-8256.19	-6153.84	-8899.98	-6812.88	-8909.08	-6841.86

Notes: The sample period extends from January 1990 to December 2009, resulting in 5218 daily observations. The GARCH, exponential GARCH, and GJR-GARCH models are estimated under the extended versions, where the conditional variance includes lagged values of implied volatility. ARCH terms and lags are added to account for any remaining ARCH effects and serial correlation in the residuals. These models are estimated using Bollerslev-Wooldrige robust standard errors and covariances. The figures in brackets represent *p*-values associated with the regression coefficients. LB represents the *p*-values associated with Ljung-Box tests of serial correlation in squared residuals up to the 10^{th} order. LM reports the *p*-values associated with the Lagrange Multiplier test for any remaining ARCH effects. Both tests are distributed as χ^2 on the null. LL is the maximum log-likelihood function.

Table 4. Out-of-sample forecasting accuracy of implied volatility

Model Specification	Mean Absolute Errors		Root Mean Squared Errors		Sign Bias	
	Nikkei225	S&P500	Nikkei225	S&P500	Nikkei225	S&P500
Panel A	Pre-crisis forecast period: 1990 ~ 2006					
Implied volatility	4.838	3.880	6.442	5.345	0.554	0.642
GARCH models						
GARCH	5.783	4.720	7.429	6.398	0.616	0.638
GARCH-IV	5.452	4.025	6.912	5.850	0.659	0.419
Exponential GARCH						
EGARCH	5.734	4.264	7.329	5.687	0.601	0.640
EGARCH-IV	5.890	3.986	7.609	5.764	0.642	0.436
GJR-GARCH						
GJR-GARCH	5.874	4.234	7.596	5.763	0.611	0.621
GJR-GARCH-IV	5.686	3.965	7.423	5.785	0.640	0.450
Panel B	Crisis forecast period: 2007 ~ 2009					
Implied volatility	8.582	7.604	15.713	13.330	0.645	0.503
GARCH models						
GARCH	9.994	8.158	16.857	13.120	0.565	0.528
GARCH-IV	9.606	8.172	16.499	14.229	0.661	0.460
Exponential GARCH						
EGARCH	8.998	7.540	15.552	12.474	0.523	0.409
EGARCH-IV	9.942	7.800	16.430	13.536	0.631	0.449
GJR-GARCH						
GJR-GARCH	9.808	7.454	16.159	12.570	0.557	0.552
GJR-GARCH-IV	9.352	7.218	15.905	12.173	0.651	0.457

Notes: The out-of-sample forecasts are obtained for the period from 2000 to 2009 based on the conditional volatility from the various GARCH models described by equations (6) to (9). The initial estimation period extends from January 1990 to December 1999 and out-of-sample static forecasts are obtained for the first six months of 2000. The subsequent estimation period starts from July 1990 to June 2000 and the results are used to forecast volatility over the second half of 2000. This process is repeated until the total forecast period extends to December 2009. This forecast period covers the U.S. credit crisis, which is broadly defined over the three years from 2007 to 2009. Higher-order GARCH models and additional lags are considered in order to account for the remaining ARCH effects and serial correlation. The conditional standard deviations are annualized and expressed in percentage terms for the purpose of comparison with the forecasting performance of implied volatility. Realized volatility is defined as the annualized standard deviations of squared returns over the rolling period of 30 days.